Measuring the Mutual Interaction between Coaxial Cylindrical Coils with the Bode 100

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Abstract: The mutual interaction between two coils in close proximity significantly influences the individual coils. With the Bode 100 vector network analyzer from OMICRON Lab, we measure the mutual inductance and skin- & proximity losses for coaxial cylindrical coils. The results are set into contrast with theoretical models based on classical electrodynamics. Our main results are the coupling factor dependence on the separation between the coils and the coil resistance increases due to skin- and proximity effects.

Keywords: Coupling factor, mutual inductance, skin effect, proximity effects

Physical quantities
μ .... Magnetic Permeability
σ .... Conductivity
I .... Current
r .... Filament radius
h .... Filament separation
δ........ Skin depth

ω....... Angular frequency
L........ Self-inductance
M....... Mutual-inductance
R .... Resistance
k........ Coupling factor
d........ Conductor diameter

Mathematical quantities
J_1 .... First kind Bessel function of first order

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1 Introduction

In this article we study the behavior between two coupled coaxial cylindrical coils in close proximity. Therefore, we will use a simple cylindrical coil as our device under test (DUT). A simple lumped element model which describes the interaction between two DUTs is then introduced. The resistance, inductance and the mutual coupling components of the lumped element circuit are characterized with the Bode 100 vector network analyzer (VNA). For terminology, we will distinguish between two cases:

i. a coil is placed in free space and only interacts with itself (primary coil)
ii. a secondary coil interacts with the primary coil

Case one leads to the skin effects, whereas the latter leads to the proximity effects. For simplicity, we use two identical coils (primary and secondary). In the first section of the measurement chapter, the inductance of the primary coil is directly measured with the impedance mode of the Bode 100 VNA. We will then continue with the same setup to measure the coil resistance due to skin effects. Finally, the mutual interaction between the primary and secondary is characterized. Therefore we measure the transfer function which for our simple lumped element model yields to the mutual inductance and the coupling factor. Furthermore, the implications of the mutual interaction on the coil resistance are investigated. The results of skin- and proximity resistance as well as the mutual inductance are compared with the theory of [refB].

1.1 Coil assembly

The DUT coil we will use for the measurement section is a simple self-assembled coil, consisting of a 1mm copper wire wrapped around a cylindrical plastic shape. The geometrical data is given in table 1 and a picture of the assembly in figure 1. The coils are mounted to a BNC jack, compatible to the Bode 100 cabling. Note that two identical coils were build, denoted as the primary and secondary coil.

Coil radius r:	6.5 mm
Wire diameter d:	1 mm
Layers:	2
Windings:	9
Conductivity σ:	$5.96 \times 10^7 S/m$

Table 1: Coil configuration data

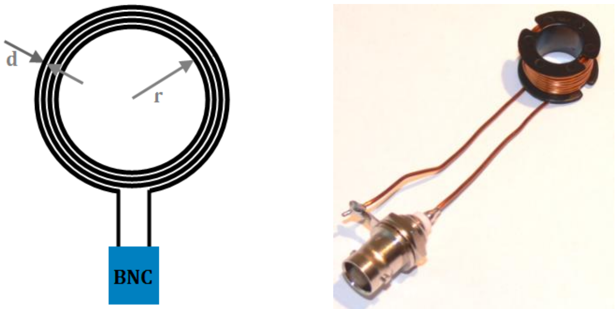

Figure 1: (a) Layout and (b) picture of the assembled coil

1.2 Lumped element model

Based on the coil assembly in section 1.1, we can set up a theoretical model which describes the frequency behavior of the coupled primary and secondary coil for low frequencies, i.e. $f \leq 10$ MHz. Since the wavelength of low frequency signals is much larger than the coil dimensions and wire length, a lumped element approach is appropriate. Figure 2 shows a schematic model of the two interacting coils

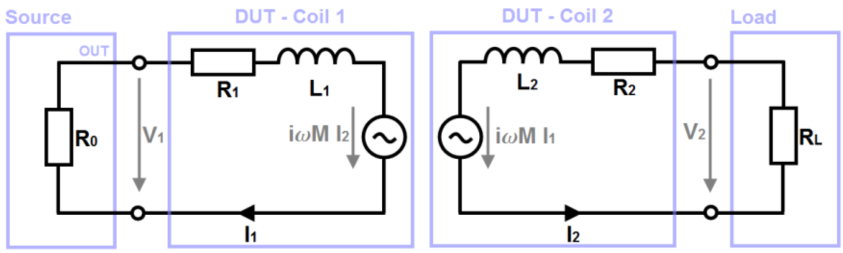

Figure 2: Equivalent circuit for the transfer function measurement

Here, R_1 and R_2 represent the individual coil resistances and L_1 and L_2 the coil inductances. The coupling is introduced by the mutual inductance M. The source resistance R_0 is 50 Ω (Bode 100 source impedance) and the load resistance R_L is set to 10 Ω. Based on this lumped element model, we can write down a set of equations, derived by Kirchhoff's circuit laws:

$$V_1 = (R_1 + i\omega L_1)I_1 + i\omega M\, I_2 \quad (1)$$

$$V_2 = (R_2 + i\omega L_2)I_2 + i\omega M\, I_1 \quad (2)$$

Note that $\omega = 2\pi f$, where f is frequency of an AC voltage applied to the circuit. From equations (1) and (2) we can derive the transfer function V_2/V_1 by algebraic manipulation:

$$\frac{V_2}{V_1} = \frac{i\omega M R_L}{(R_1 + i\omega L_1)(R_2 + R_L + i\omega L_2) + \omega^2 M^2} \quad (3)$$

The VNA Bode 100 can measure this voltage ratio as a function of frequency. For certain frequencies, the transfer function can then be used to estimate the mutual inductance and the coupling factor between the coils. The procedure will be explained in the measurement chapter.

2 Measurements

2.1 Coil inductance

An important parameter of the lumped element model are the self-inductances L_1 and L_2. Both are measured with the impedance mode of the Bode 100. The setup simply connects the DUT with the output as indicated in figure 3 and the configuration is listed in table 2. The measurement result is given in figure 4.

Figure 3: Setup for the self-inductance measurement

Start Frequency:	10 kHz
Stop Frequency:	10 MHz
Sweep Mode:	Logarithmic
Number of Points:	401
Level:	0.00 dBm
Receiver Bandwidth:	30 Hz
Calibration:	Impedance On
Meas. Format	Inductance Ls

Table 2: Bode 100 configuration (self-inductance meas.)

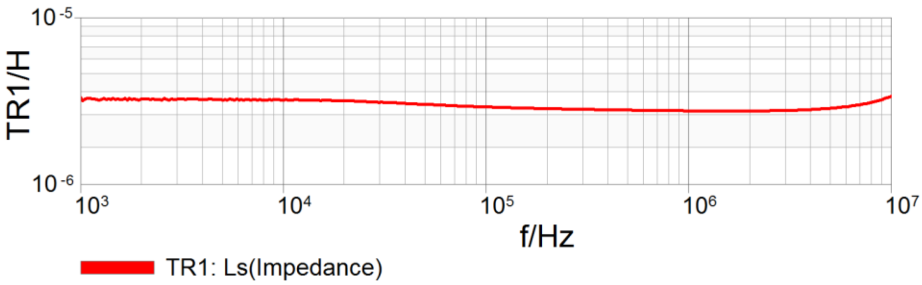

Figure 4: Inductance measurement

From the measurement result we can conclude that the coil self-inductance only changes slightly for frequencies higher than 20 kHz. The low-frequency results for the primary and secondary coil are:

primary coil L_1	3.64 µH
secondary coil L_2	3.62 µH

Table 3: Coil self-inducance for low frequencies

For simplicity, we will neglect the slight changes in the self-inductance for higher frequencies and work with the values of table 3.

2.2 Coil resistance

As a next step, we characterize the resistances R_1 and R_2 of our lumped element model. For DC currents, the coils have ohmic losses due to the finite conductivity of copper. This finite conductivity yields to a resistance, only dependent on the coil geometry. However for AC currents, the coil resistance is affected by skin- and proximity effects. A conductor carrying a time varying current will induce magnetic fields. The back action induces eddy currents which are by Lenz's law opposite to the initial current. This is the so called skin effect which can be measured directly by the Bode 100. In the following, we will focus on the frequency dependent change of the coil resistance between 1 kHz and 10 MHz. The setup is the same as for the inductance measurements (impedance mode). The only change required is to set the measurement format from "Inductance Ls" to "Resistance Rs". The result is shown in figure 5, where we also compare it to the theoretical model of [refB].

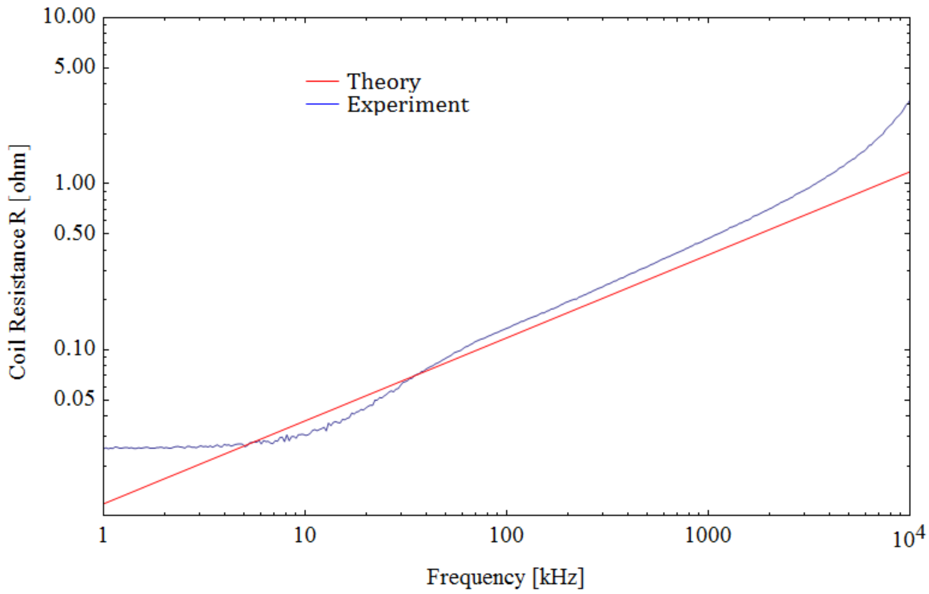

Figure 5: Skin resistance of the primary coil

In short, the effective resistance of the coil can be well described by the skin depth δ and the geometric dimensions of the coil as listed in section 1.1:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad R_{skin} \approx \frac{2}{\mu\sigma\delta d} L \tag{4}$$

Theory and experiment show a very good accordance within the range from 100 kHz to 1MHz. The deviation of approximately $\approx 0.025\Omega$ is due to the finite DC resistivity which is not regarded in the theoretical model of equation (4). The divergence for higher frequency is due to resonance effects which are no considered in the lumped element model.

2.3 Transfer function measurement and mutual inductance

As explained in the introduction, we are interested to measure the voltage ratio V_2/V_1 of the lumped element model of figure 2. Therefore, we set up the Bode 100 according to table 4 and connect the coils as indicated in figure 6.

Start Frequency:	10 Hz
Stop Frequency:	10 MHz
Sweep Mode:	Logarithmic
Number of Points:	401
Level:	0.00 dBm
Attenuator CH1 / CH2:	0 / 20 dB
Impedance CH1 / CH2:	1M / 1M Ω
Receiver Bandwidth:	30 Hz
Meas. Format	Gain (dB)

Table 4: Bode 100 configuration (transfer function)

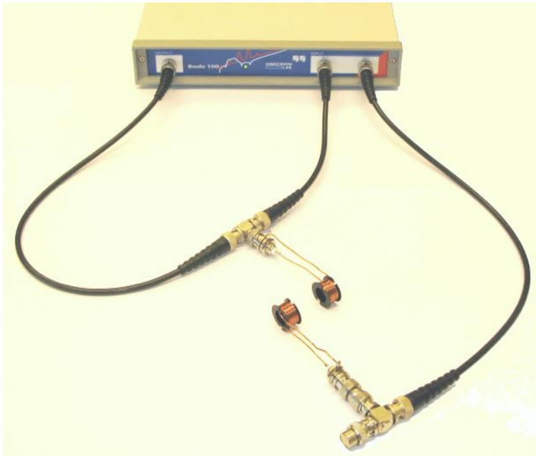

Figure 6: Setup for the transfer function measurement

The measurement result for a gap of $h = 0$ between the two coils is shown in figure 7. The transfer function has an almost flat dependence between 5 and 200 kHz which can be used to evaluate the mutual inductance and the coupling factor.

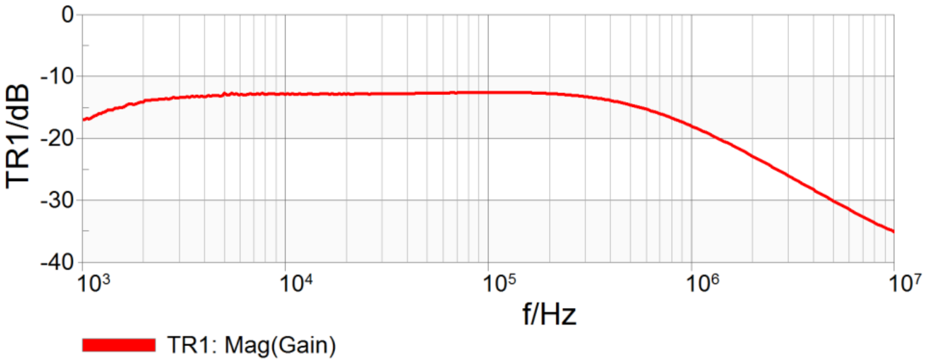

Figure 7: Measured transfer function (gap h=0mm)

Typical coil resistances $R_{1,2} \leq 0.1\ \Omega$ and $L_{1,2} \approx 3.64\ \mu H$ for frequencies smaller than 10 kHz observed in this article yield to $R_L \gg R_{1,2}$ and $R_L \gg \omega L_{1,2}$. Thus we can neglect some terms of equation (3) which explains the flat plateau of the transfer function:

$$\frac{V_2}{V_1} \approx \frac{i\omega M R_L}{i\omega L_1 R_L} = \frac{M}{L_1} \quad (5)$$

If we introduce the coupling factor $k = M/\sqrt{L_1 L_2}$, we can get

$$\frac{V_2}{V_1} \approx k\sqrt{\frac{L_2}{L_1}} \quad (6)$$

In summary, we can obtain the mutual inductance and the coupling factor from the transfer function plateau if we use $L_1 \approx 3.64\ \mu H$ and $L_2 \approx 3.62\ \mu H$. We can repeat the measurement procedure for a selection of various gaps between the coils:

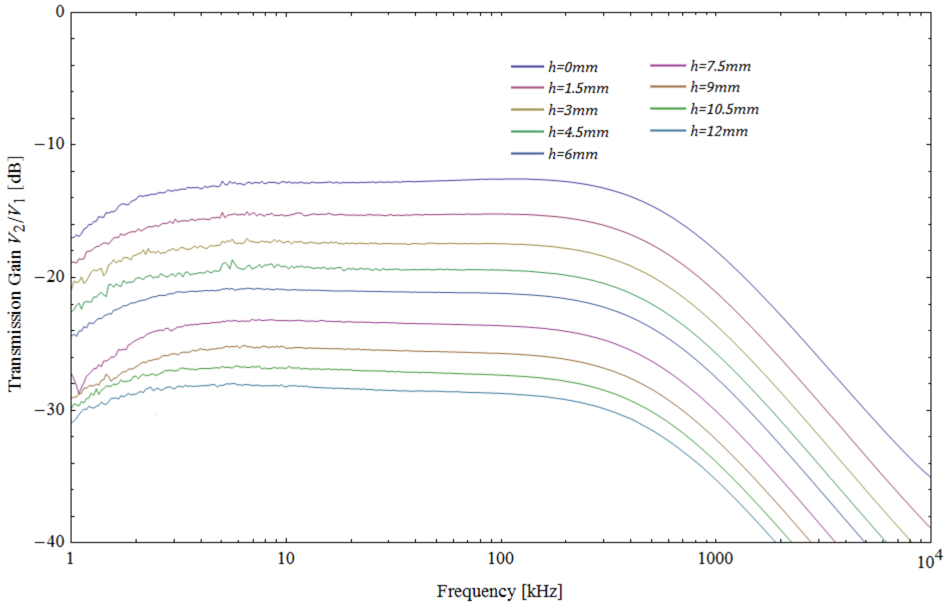

Figure 8: Transmission function for different gaps

2.4 The coupling factor

Before we extract the mutual inductance and coupling factor from the measurement results of figure 8, the theoretical model used for comparison is introduced. According to [refA], a theoretical approximation for the mutual induction between two coaxial cylindrical coils is

$$M \approx \pi \mu \bar{r}_p \bar{r}_s N_p N_s \int_0^{10^4} dm\, J_1(m\bar{r}_p) J_1(m\bar{r}_s) e^{-m|\bar{h}_p - \bar{h}_s|}, \qquad (7)$$

where we used the following parameters for the calculation:

$\mu = 4\pi\, 10^{-7}\ H/m$	magnetic permeability
$\bar{r}_p = \bar{r}_s = 7.5\ mm$	mean coil radius
$\lvert \bar{h}_p - \bar{h}_s \rvert = 10\ mm + h$	mean coil separation
$N_p = N_s = 18$	number of turns

Table 2: Parameters for the theoretical model

Note that h is the gap between the coils. It is possible to calculate the mutual inductance M from the measured coupling factor k with equation (6). L_1 and L_2 are the self-inductances (see Table 3).

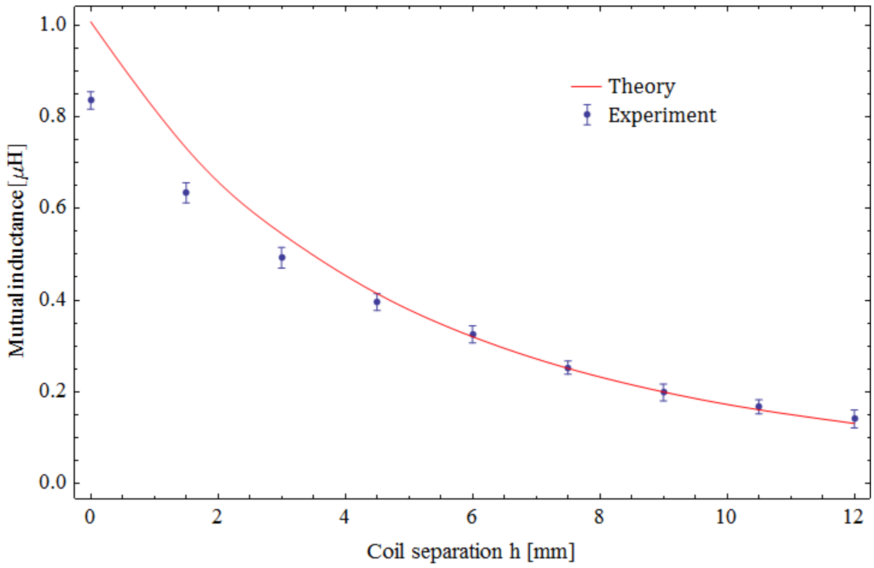

Figure 9: Mutual induction between two coils for different gaps h

Figure 6 shows a comparison of the theoretical model and experimental data (with error bars). For small coil separations, theory and experiment diverge. This is due to the fact, that for small gaps, close coil filaments contribute more to M. A more accurate model of the mutual inductance can be obtained by use of Lyle's method [refC] or other approaches of [refD]. The data points of figure 9 and the relative error to the theoretical calculation is listed here:

h [mm]	Theory	Experiment		rel. Error
	M [µH]	k	M [µH]	of M [%]
0	1.008	0.2291	0.835	20.72
1.5	0.732	0.1738	0.633	15.64
3	0.545	0.1349	0.492	10.77
4.5	0.414	0.1084	0.395	4.81
6	0.320	0.0891	0.325	1.54
7.5	0.251	0.0691	0.252	0.40
9	0.199	0.0543	0.198	0.51
10.5	0.161	0.0457	0.167	3.60
12	0.130	0.0385	0.140	7.14

Table 5: Mutual inductance and coupling factor for varying coil separation

2.5 Coil resistance due to proximity effects

If we now consider the two coupled coils in close proximity, the coil resistance will be affected significantly. The mutual coupling, which has the same physical origins as the skin effect, leads to a so called proximity effect. To measure the mutual influence of the coils, we keep the resistance measurement setup from section 2.2, but bring a secondary coil in axial alignment in proximity to the primary coil. Then the resistance for the primary coil is measured for different gaps. Note that for large gaps the coil resistance should be equal to the skin resistance of figure 5.

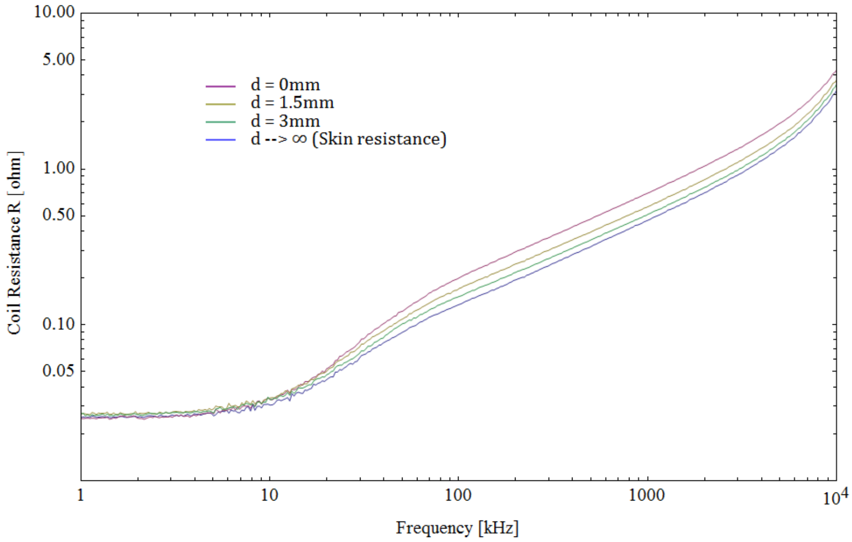

Figure 10: Coil resistance for varying coil separations

For coils with a wire diameter of about 1 mm, the model of [refB] for skin and proximity effects is valid for frequencies higher than 100 kHz. The theoretical model for the resistance is given by

$$R_{prox} \approx \frac{2}{\mu\sigma\delta d}\left(L + \frac{I_2}{I_1}M\right). \tag{8}$$

For no coil in close proximity, i.e. $I_2 = 0$ or $M = 0$, the resistance reduces to the skin effect. In order to demonstrate the proximity effect model of equation (8), we normalize the results of figure 11 by the skin resistance, i.e. we plot R/R_{skin}:

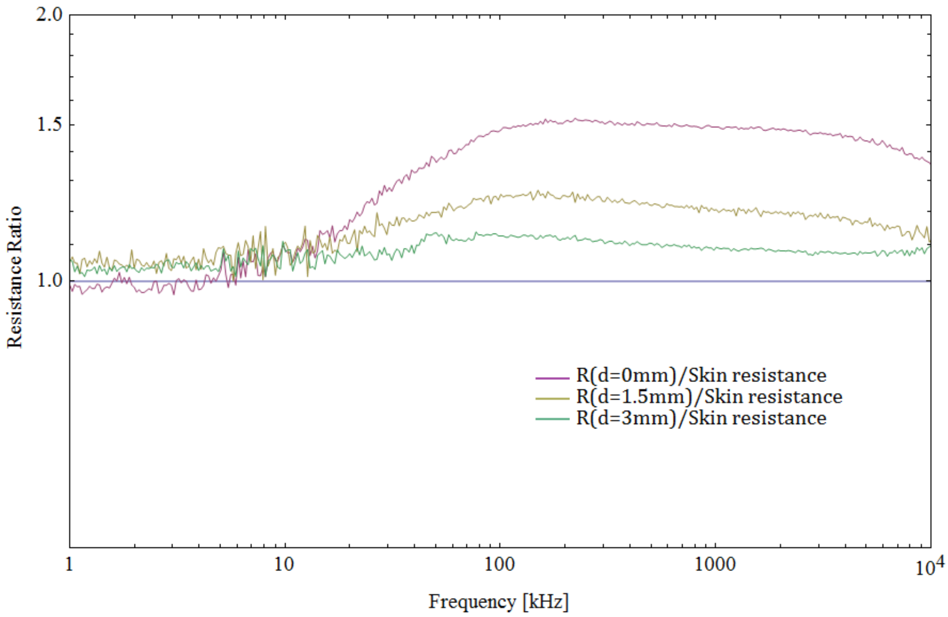

Figure 11: Normalized coil resistance

In detail, this graph shows

$$\frac{R_{prox}}{R_{skin}} \approx \left(1 + \frac{I_2}{I_1}\frac{M}{L}\right). \tag{5}$$

From the flat plateaus we can thus extract the ratio of $I_2 M/(I_1 L)$ and compare it to the theoretical model of M in equation (7), where $I_1 = V_1/R_0$ and $I_2 = V_2/R_L$.

h [mm]	Measured resistance ratio	Theoretical resistance ratio	relative Error [%]
0	1.493	1.381	8.11
1.5	1.232	1.201	2.58
3	1.121	1.112	0.81

Table 6: Experimental and theoretical resistance ratio for different gaps

3 Summary

In summary, we demonstrated how to measure the inductance, resistance due to skin- and proximity effects and the mutual coupling between coaxial cylindrical coils with the vector network analyzer Bode 100. The measurement results were set in contrast with theoretical models based on classical electrodynamics and showed a good agreement. A potential application of this article is the optimization of wireless power link efficiency for cylindrical coils.

References

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[refC] F. W. Grover: Inductance Calculations: Working Formulas and Tables, Dover Publications (1946), p.88-90

[refD] Lyle: Inductance Calculations: Working Formulas and Tables, Dover Publications (1946), p.88-90