



## **Laboratory 2**

# **Delta-Sigma Modulation**

**The Art of Oversampling, Noise Shaping and Averaging**

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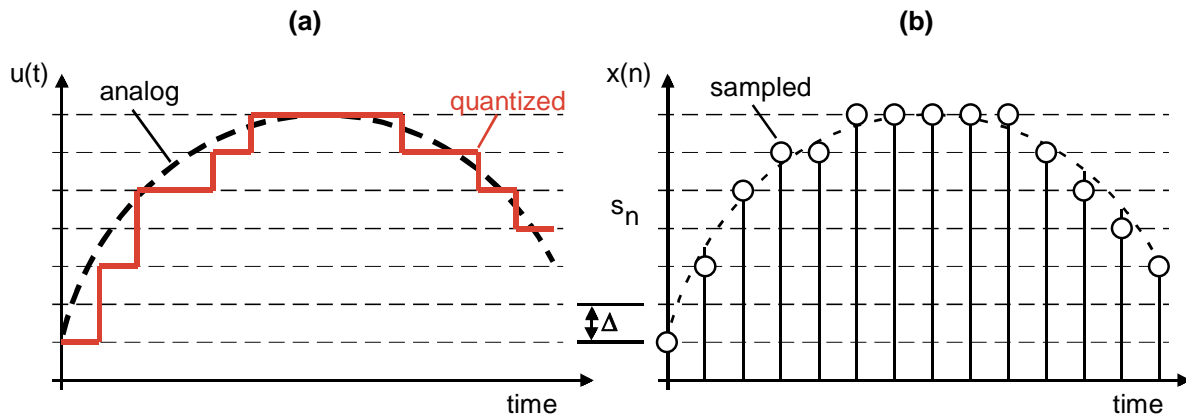
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**Abstract.** Quantization noise will be investigated in unmodulated and  $\Delta\Sigma$  modulated form. Mathematical roots to understand the matter are discussed before practical evidence is given. Quantization noise power within the baseband  $0..f_B$  is proportional to  $\Delta^2/OSR^{2L+1}$ , with  $\Delta$  being the quantizer's minimum step,  $OSR=f_s/2f_B$  the oversampling ratio,  $f_s$  the sampling frequency and  $L$  the modulator's order.

## 1 Introduction



**Fig. 1:** (a) Analog and quantized signal, (b) sampled digital signal

**Any signal real is noisy.** Mostly we try to reduce digital noise by a higher bit-width of the processed numbers. The  $\Delta\Sigma$  modulator goes the opposite direction: It translates a long bit-vector or an analog quantity into a fast data stream with few parallel bits. The data stream encodes the input value within its average. This requires quantization [1]-[4].

**Quantization** is inevitable for analog-to-digital (A/D) conversion. Quantization and quantization noise correspond to the mathematical process of rounding and round-off noise. The minimum step of quantization is termed  $\Delta$ . It can be a bit, a voltage, a current, etc.

**The power of noise shaping** demonstrates Table 1. To improve the signal-to-noise ratio of the averaged signal by 60dB (= factor 1000 in voltage,  $10^6$  in power or ca. 8 bit signal width) an unmodulated signal must be  $OSR=10^6$  times faster than required acc. to Nyquist. For a 1<sup>st</sup> order modulator an  $OSR$  of 126 is enough and for a 2<sup>nd</sup> order modulator an  $OSR$  of only 27! (PS: This calculation assumes that there are no other noise sources than quantization noise.)

**Table 1:**  $OSR$  is the oversampling ratio and  $L$  the modulator's order. Data was taken from [3].

SNR <sub>dB</sub>	20 dB	40 dB	60 dB	80 dB	100 dB
$OSR @ L=2$	5	11	27	67	168
$OSR @ L=1$	6	28	126	578	2657
$OSR @ L=0$	100	10322	1,05E6	1,06E8	1,08E10

**The theoretical part** of  $\Delta\Sigma$  modulation is relatively difficult [1]-[4]. But it is of high practical importance and a useful device to demonstrate numerous signal processing theorems.

**The practical part** of this laboratory is relatively easy and many results are more or less given. Some documents help to understand equipment [5]-[8] and control loops [9].

**The organization of this laboratory** is as follows:

Section 2 presents some very fundamental theoretical background on signal processing which most students should already know.

Section 3 presents particular statements on  $\Delta\Sigma$  modulation.

Section 4 offers experimental verification.

Section 5 draws relevant conclusion.

## 2 Some Fundamentals on Signal Processing

### 2.1 Correlation

- Correlated signals depend on each other
- Uncorrelated signals do not depend on each other

Correlated signals sum in amplitude:  $y_{sum,corr} = x_1 + x_2 + x_3 + \dots + x_N$  (2.1)

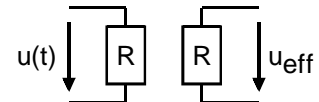
Uncorrelated signals sum in power:  $y_{sum,uncorr} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots + x_N^2}$  (2.2)

Different frequencies are always uncorrelated. (2.3)

### 2.2 Root-Mean-Square Computation of Effective Amplitudes

#### 2.2.1 Effective Voltages and Currents and their RMS Computation

Fig. 2.2.1: The effective DC voltage  $u_{eff}\{u(t)\}$  dissipates the same power at a resistor R like the  $u(t)$ .



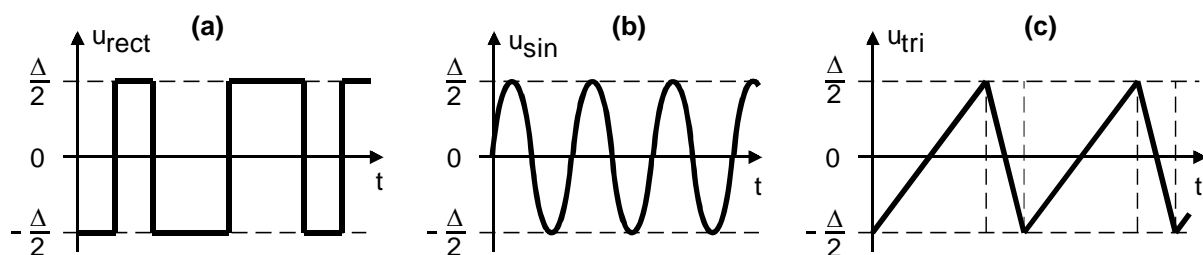
The effective DC value of voltage  $u(t)$  is termed  $u_{eff}\{u(t)\}$ . It dissipates the same power at a resistor R as the  $u(t)$ . Good voltmeters either heat a reference resistor to the same temperature as  $u(t)$  or compute  $u_{eff}$  by the **Root Mean Square** (rms) method: Square a signal, average it over time (mean) and then compute the square root of it.

Time continuous:  $u_{eff} = \sqrt{u^2} = \sqrt{\frac{1}{T} \int_0^T u^2(t) dt}$  (2.4)

Time discrete for samples  $x_i$ ,  $i=1\dots N$ :  $x_{eff} = \sqrt{x^2} = \sqrt{\frac{1}{N} \sum_{i=1}^N x_i^2}$  (2.5)

Note: The frequently observed expression  $\bar{u}^2$  is 0 for an AC signal, because its mean value is 0 and  $0^2=0$ , too. Correct is averaging after squaring:  $u_{eff}^2 = \bar{u^2}$ .

#### 2.2.2 Effective Values of Some Particular Waveforms



**Fig. 2.2.2:** Particular waveforms: (a) rectangular, (b) sinusoidal, (c) triangular.

**Fig. 2.2.2** shows (a) a rectangular, (b) a sinusoidal und (c) a triangular signal oscillating between the values  $\Delta/2$  and  $-\Delta/2$ . Its total power at  $1\Omega$  and its effective voltage are given by

$$\text{Rectangular:} \quad \overline{u_{rect}^2} = \frac{(\Delta/2)^2}{1} = \frac{\Delta^2}{4} \quad \leftrightarrow \quad u_{rect,eff} = \frac{\Delta}{2} = \frac{\Delta/2}{\sqrt{1}} \quad , \quad (2.6)$$

$$\text{Sinusoidal:} \quad \overline{u_{sin}^2} = \frac{(\Delta/2)^2}{2} = \frac{\Delta^2}{8} \quad \leftrightarrow \quad u_{sin,eff} = \frac{\Delta}{\sqrt{8}} = \frac{\Delta/2}{\sqrt{2}} \quad , \quad (2.7)$$

$$\text{Triangular:} \quad \overline{u_{tri}^2} = \frac{(\Delta/2)^2}{3} = \frac{\Delta^2}{12} \quad \leftrightarrow \quad u_{tri,eff} = \frac{\Delta}{\sqrt{12}} = \frac{\Delta/2}{\sqrt{3}} \quad . \quad (2.8)$$

Adding an offset: The offset voltage is frequency 0Hz. Adding in power delivers

$$\text{Combined:} \quad \overline{u_{comb}^2} = \overline{u_{offset}^2} + \overline{u_{xxx}^2} \quad \leftrightarrow \quad u_{comb,eff} = \sqrt{\overline{u_{offset}^2} + \overline{u_{xxx,eff}^2}} \quad (2.9)$$

with xxx standing for *rect*, *sin*, *tri* or others.

## 2.3 Estimating the Total Power of Quantization Noise

Quantization is the process of expressing quantities as integral multiples of a minimum step, also termed  $\Delta$  in signal processing. Quantization corresponds to the mathematical process of rounding and the quantization error  $e_q$  corresponds to the mathematical round-off error.

### 2.3.1 Worst Case: Signal Power $\ll$ Quantization Noise Power

**Fig. 2.3.1:** Averaged analog signal  $u(t)$  is tiny compared to the quantizer's  $\Delta/2$ .

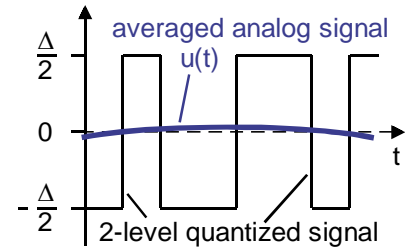


Fig. 2.3.1 illustrates the worst case scenario for a 2-level (1-bit) quantizer and a signal  $u(t)$  obtained from the bit-stream by averaging. This case is of very high practical relevance!

$$\text{Quantization noise is obtained from} \quad \boxed{e_q(t) = u_{ideal}(t) - u_{quantized}(t)} \quad . \quad (2.10)$$

It is also clear that in the worst-case (2.10) becomes in the limit of  $u(t) \rightarrow 0$

$$\text{If } |u(t)| \ll \Delta/2 \rightarrow \quad e_q(t) \approx -u_{quantized}(t) \quad . \quad (2.11)$$

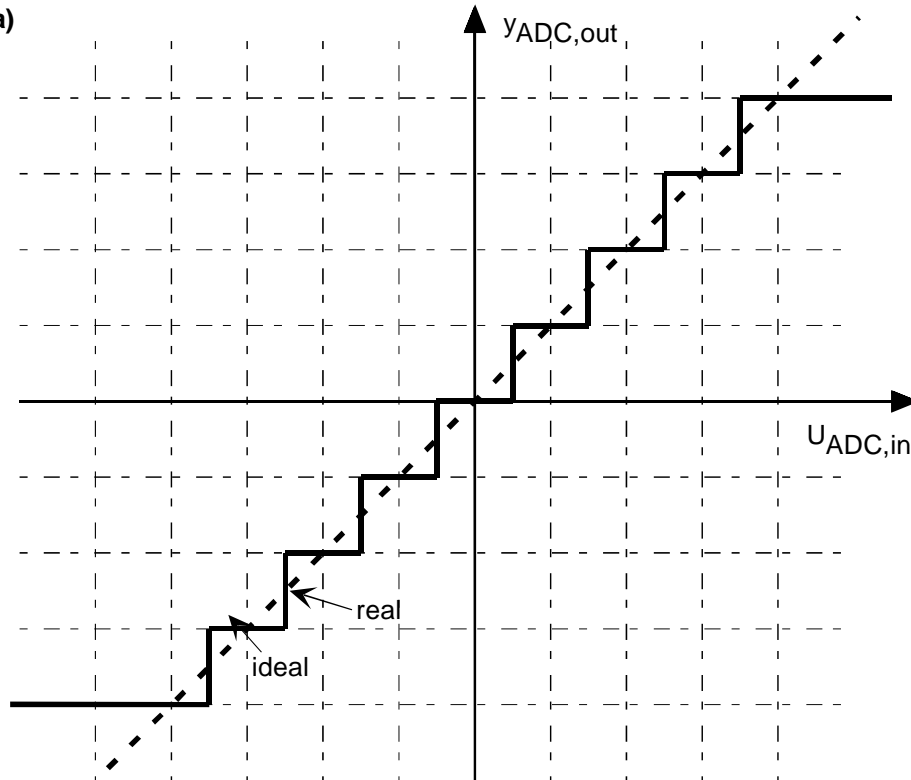
The total noise power and effective noise voltage is then given by (2.6):

$$\text{Worst Case: } |u(t)/\Delta| \rightarrow 0 : \quad \boxed{\overline{e_{n,q,worst}^2} \xrightarrow{|u(t)/\Delta| \rightarrow 0} \frac{\Delta^2}{4}} \quad \leftrightarrow \quad \boxed{e_{n,q,worst,eff} \xrightarrow{|u(t)/\Delta| \rightarrow 0} \frac{\Delta}{2}} \quad .(2.12)$$

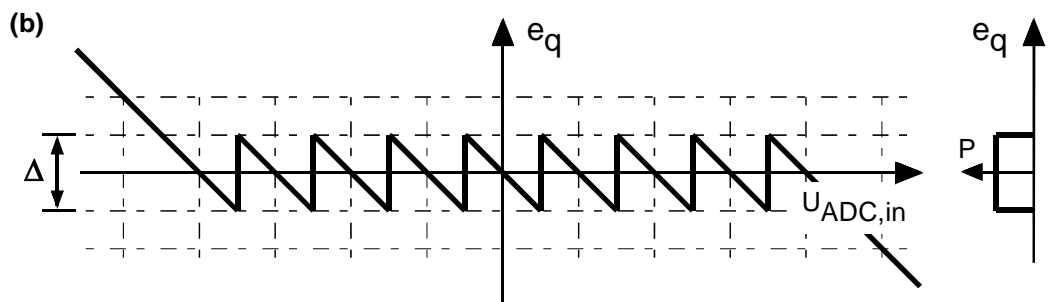
### 2.3.2 Best Case: Quantization Noise is Equidistributed Over Interval $\Delta$

**Fig. 2.3.2:** Illustrating the best-case quantization situation

(a) ideal and real characteristics of a quantizer



(b) left : quantization noise  $e_q$



(c) right:  $e_q$  equidistributed over  $\Delta$ .

From signal processing literature [1]-[4] we know (without proof) that

- If the input signal is sufficiently busy (no DC!) and (2.13)
- the quantization error  $e_q(t)$  is equidistributed over the interval  $\Delta$  (2.14)

then it can be shown [1]-[4], that the total noise power is described by (2.8), the effective value of a triangular oscillation:

**Best Case:**  $e_q$  equidistributed over  $\Delta$  : 
$$\overline{e_{n,q,best}^2} = \frac{\Delta^2}{12} \leftrightarrow e_{n,q,best,eff} = \frac{\Delta}{2\sqrt{3}} . \tag{2.15}$$

The reality will be somewhere between worst and best case, i.e. 
$$\frac{\Delta^2}{12} \leq e_{n,q}^2 \leq \frac{\Delta^2}{4} . \tag{2.16}$$

## 2.4 Bel and Dezibel

In honor of Graham Bell a factor 10 in signal power is termed a Bel, and 1 B = 10 dB, just as 1 m = 10 dm or 1 liter = 10 dl. As power corresponds to square of amplitude ( $p=u^2/R=i^2 \cdot R$ ) and  $\log(x^2)=2 \cdot \log(x)$  we get

$$\text{Signal-Ratio} = \log_{10} \frac{p_2}{p_1} B = 10 \log_{10} \frac{p_2}{p_1} dB = 20 \log_{10} \frac{u_2}{u_1} dB = 20 \log_{10} \frac{i_2}{i_1} dB . \quad (2.6)$$

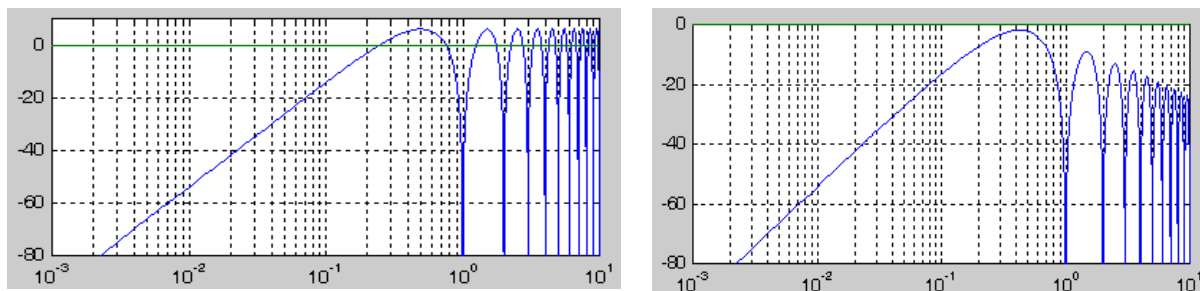
**Question:** 10dB is what factor in signal power? 10dB is what factor in effective voltage?

**1 B = 10dB is a factor 10 in power → a factor  $\sqrt{10}$  in voltage.**

## 2.5 Estimating Signal Averages on Logarithmic Scale

Printing averaged values on logarithmic axis delivers values in the upper range of the signal.

- The average value of signal  $2 \sin^2(\pi F)$  shown in Fig. 2.5(a) is 1 or 0dB.
- The average value of signal  $2 \sin^2(\pi F) 1/(1+\pi F)$  in Fig. 2.5(b) is 0.1061 or -19.5dB.



**Fig. 2.5:** (a) Bode-plot of  $2 \sin^2(\pi F)$

(b) Bode-plot of  $2 \sin^2(\pi F) 1/(1+\pi F)$

## 2.6 Time-Continuous versus Time-Discrete Models

- Time-continuous systems are modeled in  $s$  (, i.e. by Laplace transformation).
- Time-discrete systems are modeled in  $z=e^{sT}$  with  $T=1/f_s$  and  $f_s$  sampling frequency.
- In both cases – time-continuous and time-discrete – transfer functions are computed by setting  $s=j\omega$ .

In signal processing we are rather interested in the relative frequencies  $F=f/f_s$  and  $\Omega = \omega / f_s$  than in the absolute frequencies  $f$  and  $\omega=2\pi f$ .

If  $H(z)$  is a transfer functions in  $z$ , then  $|H(z)|$  is periodic in  $f_s$ . It is enough to know  $|H(z)|$  in the range  $f=0 \dots 1/2 f_s$  corresponding to  $F=0 \dots 1/2$  and  $\Omega=0 \dots \pi$ . the rest is clear from symmetry considerations.

**In the time-discrete domain** we cannot represent signals at  $f > f_s/2$ . Therefore we assume the total quantization noise to be distributed within the interval  $f=0 \dots f_s/2$ .

**In the time-continuous world** we definitively have frequencies  $f > f_s/2$ .

Ideal time-continuous differentiator with transit frequency  $\omega_T$ : 
$$H_{c,int}(j\omega) = \frac{j\omega}{\omega_T} = \frac{jf}{f_T} \quad (2.7)$$

Ideal time-discrete differentiator with  $F=fT=f/f_s$ : 
$$H_{d,int}(z) = z^{\frac{1}{2}} - z^{-\frac{1}{2}} = 2j \sin(\pi F) \quad (2.8)$$

## 2.7 Frequency Range to be Measured for Sampled Signals

**Fig. 2.6:** NTF of 2<sup>nd</sup> order  $\Delta\Sigma$  modulator with a measured frequency range up to  $F=5$  corresponding to  $f=5f_s$ , with sampling frequency  $f=100\text{KHz}$ .

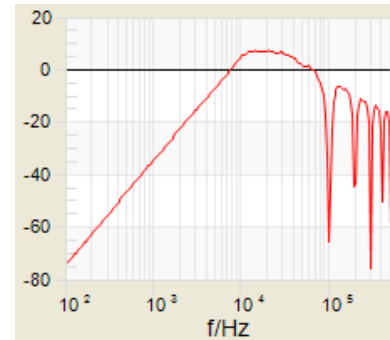


Fig. 4.2.6(d) illustrates the measured NTF of the 2<sup>nd</sup> order  $\Delta\Sigma$  modulator in the range  $f=10\text{Hz} \dots 50\text{KHz}$  for  $f_s=100\text{KHz}$ . Fig. 2.6 illustrates what happens if we extend the frequency range by a factor 10. The measurement interval was extended to 500KHz corresponding to  $5f_s$  or  $F=5$ . With constant  $C$  the measured function should be

$$|H(F)| = C \sin^2(\pi F)$$

if the output samples were Dirac pulses. Fig. 2.6. shows 5 maxima of the  $\sin^2$  - function. Caused by the finite pulse width  $\tau$  we see a  $-20\text{dB/dec}$  weighting for high frequencies, which can be approximated as

$$|H(F)| = C \sin^2(\pi F) \cdot 1/(1 + \pi F f_s \cdot \tau)$$

Like in most cases the pulse width here is  $\tau = 1/f_s$ , so that the approximation becomes

$$|H(F)| = C \sin^2(\pi F) \cdot 1/(1 + \pi F).$$

As  $\sin(x) \approx x \rightarrow \sin^2(x) \approx x^2$  for  $x \ll 1$  we see a slope of  $40\text{dB/dec}$  in Fig. 2.6 for  $f \ll 50\text{KHz}$ .

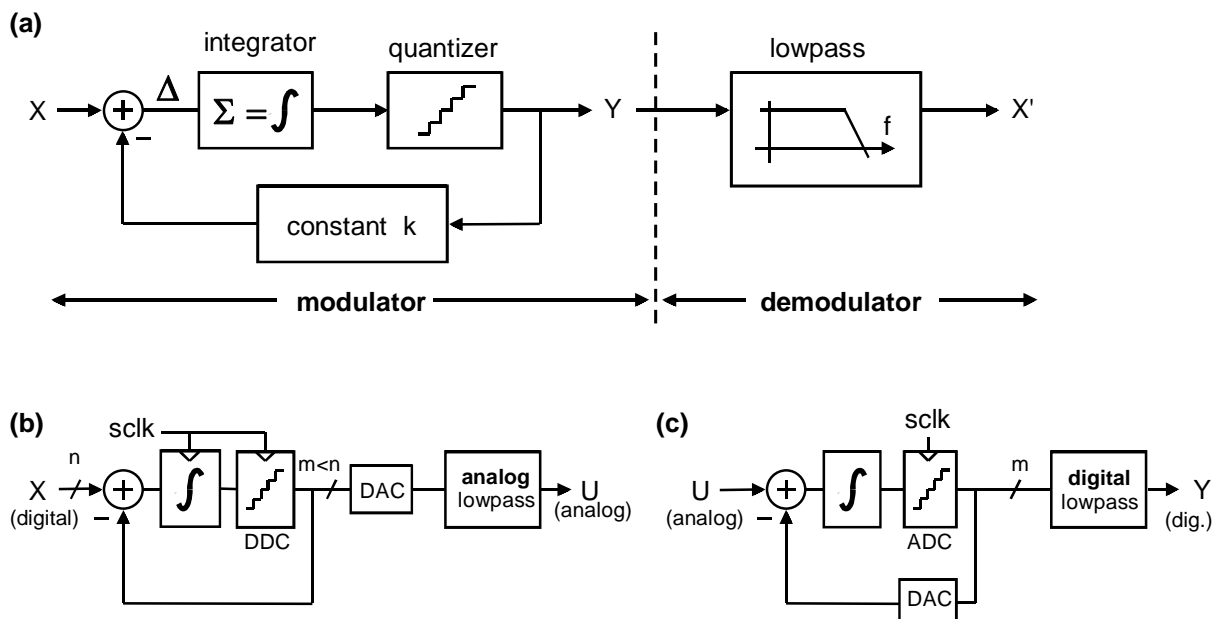
## 2.8 Check your Knowledge

How do you sum correlated signals? How do you sum uncorrelated signal? How do you sum signals with different frequencies? What are the formulas to compute effective signal values by the rms method, for time-continuous and time-discrete signals? What are effective values of rectangular, sinusoidal and triangular waveforms, with and without offset? How much is 10dB as factor in amplitude and in power? How do determine the mean value of signals on logarithmic axis? What frequency range do we observe for sampled signals?



## 3 Theory of Delta-Sigma Modulation

### 3.1 Delta-Sigma Modulation



**Fig. 3.1:** (a) Basic principle of  $\Delta\Sigma$  modulation and demodulation (b)  $\Delta\Sigma$  DAC, (c)  $\Delta\Sigma$  ADC.

**Fig. 3.1(a)** illustrates the basic principle of  $\Delta\Sigma$  modulation: A quantizer reduces the resolution of the input signal. The errors which occur are memorized by the integrator and the average output error is minimized by the feedback loop. For high loop amplification the behavior of the modulator can be approximated by  $k^{-1}$  with  $k$  being constant. To allow for averaging the output samples the modulator must sample high above the Nyquist rate, i.e. at high oversampling ratio (OSR). Demodulation is averaging the high-data rate output samples. This is done by a lowpass, which can be seen as a weighted averager.

**Fig. 3.1(b)** illustrates how to use a  $\Delta\Sigma$  modulator as digital-to-analog converter (DAC): A digital-to-digital converter (DDC) reduces the sampling rate and drives a high quality, low-resolution (often one bit) DAC. After averaging by an analog lowpass we have an accurate analog signal. The DDC is easiest realized by using some of the most significant bits of the integrator's state vector.

**Fig. 3.1(c)** illustrates how to use a  $\Delta\Sigma$  modulator as analog-to-digital converter (ADC): We use an ADC in the forward network and a DAC in the feedback branch, both DAC and ADC with low resolution (often one bit). After averaging by a digital lowpass we have an accurate digital signal.

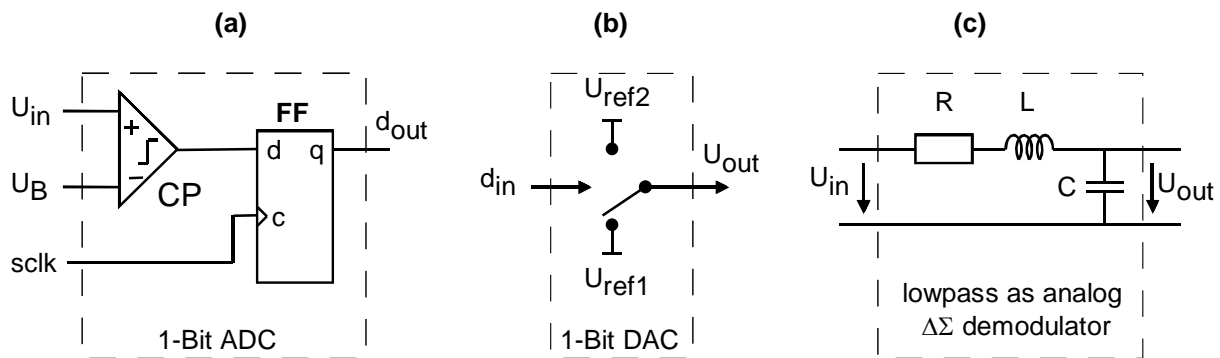
#### Check your knowledge:

What is the basic topology of a  $\Delta\Sigma$  modulator?

What is the more particular topology of a  $\Delta\Sigma$  modulator for D/A conversion?

What is the more particular topology of a  $\Delta\Sigma$  modulator for A/D conversion?

## 3.2 The Building Blocks



**Fig. 3.2:** (a) 2-level ADC. (b) 2-level DAC, (c) Lowpass as  $\Delta\Sigma$  demodulator.

The difference signal symbolized by  $\Delta$  (which is not the minimum step  $\Delta$ !) and the integrator by  $\Sigma$  are the name-giving building blocks of the  $\Delta\Sigma$  modulator. The other two important things are the quantizer and the fact, that the transfer function of the feedback branch is constant over frequency. The integrator realizes high loop gain for frequencies low compared to the sampling frequency  $f_s$ . This is important because signal- and noise transfer functions

$$STF = \frac{Y}{X} = \frac{A}{1+kA} \xrightarrow{|kA| \gg 1} k^{-1}, \quad (3.1)$$

$$NTF = \frac{1}{1+kA} \xrightarrow{|kA| \rightarrow \infty} 0 \quad (3.2)$$

require the condition  $|kA| \gg 1$  to remove the forward network  $A$  out of the formula [1]-[4]. The STF in (3.1) now depends on the feedback branch  $k$  only. As this is a DAC in Fig. 3.1(c), we can realize a good  $\Delta\Sigma$ -ADC from a poor ADC and a good DAC. As good DACs are significantly easier to realize than good ADCs this modulator transfers the quality properties of the DAC to the  $\Delta\Sigma$  ADC. (Note that  $k^{-1}$  is an ADC when  $k$  is a DAC.)

**Fig. 3.2(a)** illustrates a 2-level quantizer making the circuit simple and cheap. A/D conversion is a two-dimensional process by discretization of both value und time, realized by comparator  $CP$  and flipflop  $FF$ , respectively, in this laboratory.

**Fig. 3.2(b)** illustrates a 2-level DAC, often realized as flipflop using its output transistors as switches. It is simple, cheap, accurate and energy-efficient.

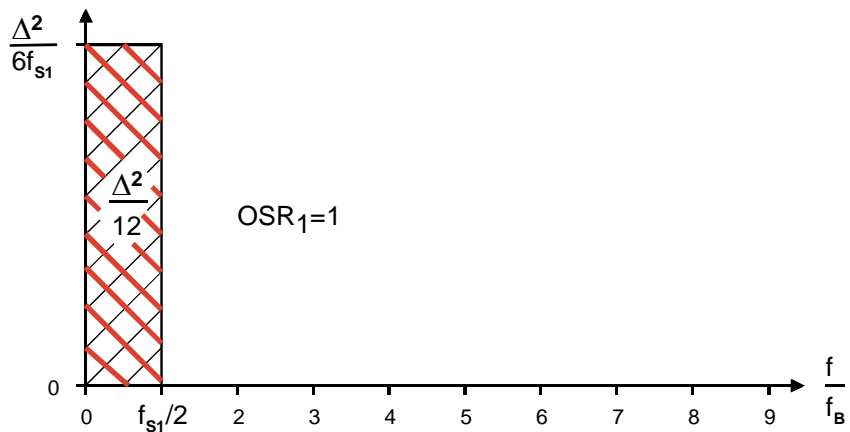
- Negative: Quantization is extremely rough.
- Strong dependence on the signal integrity of  $U_{ref1}$ ,  $U_{ref2}$ ; in case of a FF:  $V_{DD}$  and  $gnd$ .
- + DAC is 100% linear, as a line through 2 points is straight. More levels by  $\rightarrow$  DEM [1], [2].
- + Cheap: Very high energy efficiency obtainable, no need for analog devices.

**Fig. 3.2(c)** shows how easy and cheap an analog  $\Delta\Sigma$  demodulator can be realized as lowpass. Positive: Such a lowpass can always deliver an accurate average of  $U_{in}$ , independent from tolerances of  $R$ ,  $L$ ,  $C$ . Precondition is that the data rate of  $U_{in}$  is sufficiently higher than the lowpasses cutoff frequency. Again, we have trade-off between accuracy and speed. Accurate devices are a problem, but high-speed, time-precise clock signals are available in electronics.

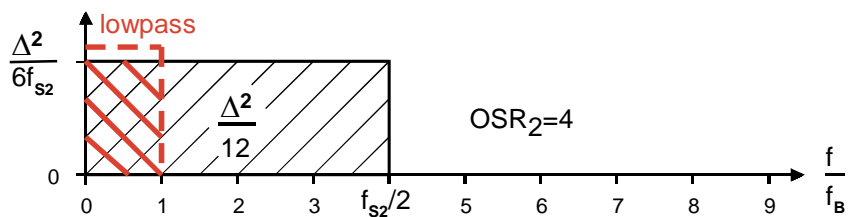
### 3.3 White Quantization Noise (0<sup>th</sup> Order Noise Shaping)

Fig. 3.3:

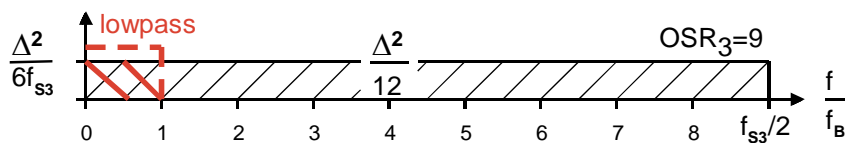
(a) no oversampling



(b) Using an OSR=4 and an ideal lowpass at  $f_B$ .



(c) Using an OSR=9 and an ideal lowpass at  $f_B$ .



A  $\Delta\Sigma$  modulator is a quantizer operated by a control loop to optimize the quantization noise profile. Understanding the process of quantization is essential!

**Definitions:**

- Sampling rate:  $f_s$  .
- Baseband:  $0 \dots f_B$  .
- Oversampling ratio:  $OSR = f_s / 2f_B$  . (3.4)

Thesis: If the input signal is sufficiently busy the total noise power is distributed with constant power density over the frequency range  $0 \dots f_s/2$ . As uncorrelated signals sum in power and different frequencies are always uncorrelated, it is noise power (not noise voltage) that equidistributes over the frequency range  $0 \dots f_s/2$ .

Fig. 3.3 illustrates these statements. A signal within the baseband  $0 \dots f_B$  is sampled with same  $\Delta$  but different sampling frequencies  $f_{s1}, f_{s2}, f_{s3}$ :

- (a)  $f_{s1} = 2f_B$  has no oversampling:  $OSR_1 = f_{s1} / 2f_B = 1$ .
- (b)  $f_{s2} = 8f_B \rightarrow OSR_2 = f_{s2} / 2f_B = 4$ . Noise power (voltage) within baseband lower by factor 4 (2).
- (c)  $f_{s3} = 18f_B \rightarrow OSR_3 = f_{s3} / 2f_B = 9$ . Noise power (voltage) within baseband lower by factor 9 (3).

Taking advantage of the  $1/f_s$ -reduction in baseband-noise power a lowpass with cutoff-frequency  $f_B$  is required. The red dashed lines in Fig. 3.3 show ideal lowpasses with cutoff-frequency  $f_B$ .

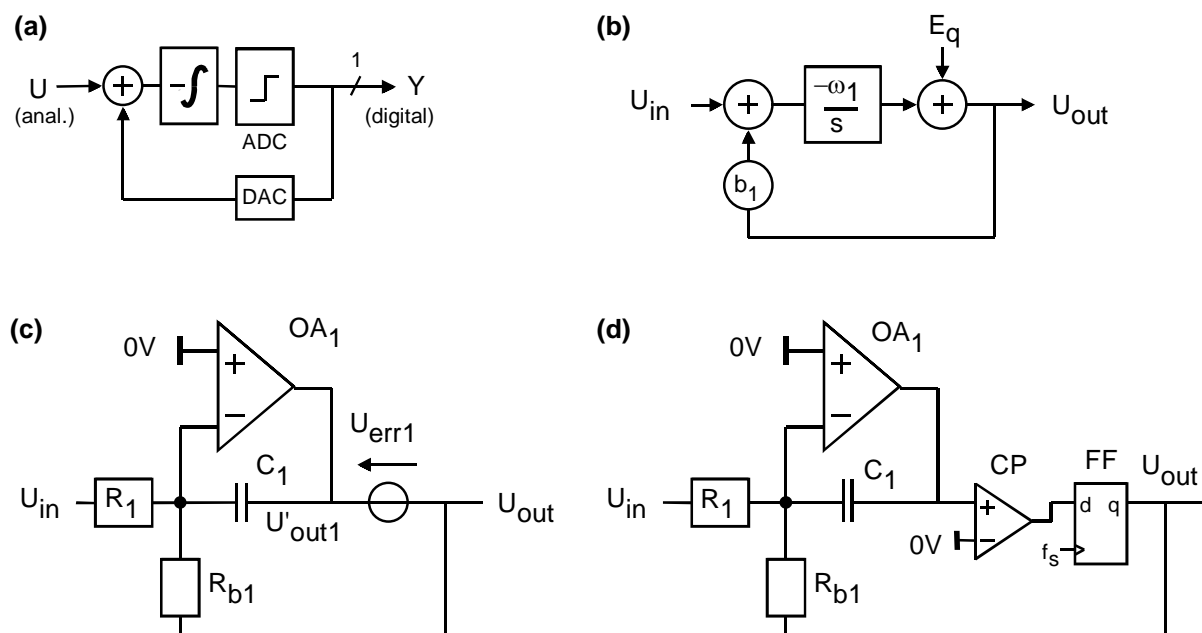
**We matter we have to check by measurement:**

- Quantization noise **power** density (in  $V^2/Hz$ ) is proportional to  $\Delta^2/f_s$ . (3.5)

- Effective quantization noise **voltage** density (in  $V/\sqrt{Hz}$ ) is proportional to  $\Delta/\sqrt{f_s}$ . (3.6)

(In data sheets we often find the acronym *rtHz* for  $\sqrt{Hz}$ .)

### 3.4 First Order $\Delta\Sigma$ Modulator (1<sup>st</sup> Order Noise Shaping)



**Fig. 3.4-1:** (a) High-level schematics, (b) depicted for modeling, (c) OpAmp realization with  $\omega_1=1/R_1C_1$ ,  $b_1=R_1/R_{b1}$  and error source  $U_{err1}$ , (d)  $U_{err1}$  replaced by a 2-level quantizer.

While the signal transfer function (STF) is flat, the noise transfer function (NTF) shapes the quantization noise according to  $1 / integrator = differentiator$ . The transfer function of an ideal time-continuous differentiator – modeled in the  $s$ -domain – is

$$|H_{s,diff}(f)| = f/f_T,$$

with transit frequency  $f_T$ . Due to the clocked quantizer we have a time-discrete situation here. Consequently, the transfer function of a differentiator – modeled in the  $z$ -domain – becomes

$$|H_{z,diff}(F)| = C_1 \cdot \sin(\pi F),$$

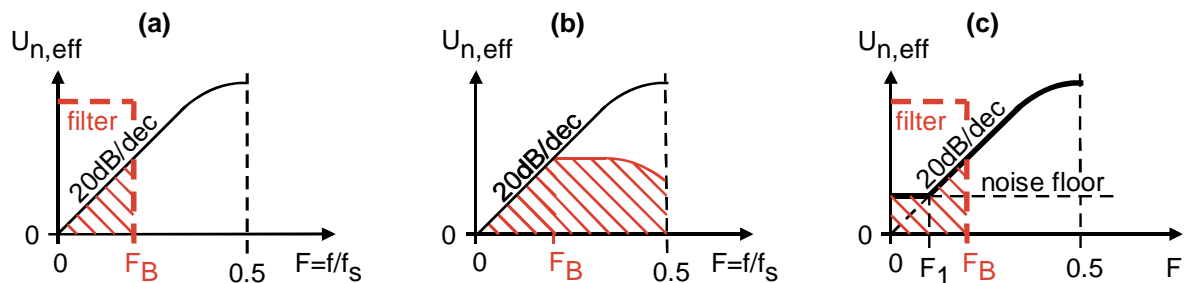
with  $C_1$  being a constant and  $F=f/f_s$ . As  $\sin(x) \approx x$  for  $x \ll 1$  we have a 20dB/dec slope in the Bode diagram for small  $F$ .

**Fig. 3.4-2(a)** sketches how quantization noise within the baseband (i.e.  $f=0\dots f_B$  corresponding to  $F=0\dots F_B$  with  $F_B=f_B/f_s$ ) is reduced compared to a situation without noise shaping.

**Fig. 3.4-2(b)** illustrates what happens, when the order of the demodulating lowpass filter is equal to the order of the modulator (given by the number of its integrators): The noise power distribution above the filter's cutoff frequency becomes constant rather than being cut off by the filter.

**Fig. 3.4-2(c)** sketches the situation with the inevitable noise floor, because there are several noise sources other than the quantizer, too. The noise in the baseband is increased compared to Fig. part (a). Red dashed lines symbolize an ideal lowpass filter.

According to the theory and in absence of other noise the noise power in the baseband for the 1<sup>st</sup> order modulator is proportional  $1/OSR^3$ ,  $\rightarrow$  eff. noise voltage is proportional  $1/OSR^{1.5}$ .



**Fig. 3.4-2:** (a) ideal situation, (b) lowpass-order=modulator-order, (c) with noise floor.

### 3.5 Second Order $\Delta\Sigma$ Modulator (2<sup>nd</sup> Order Noise Shaping)

According to the theory and in absence of other noise the noise power in the baseband for the 2<sup>nd</sup> order modulator is proportional  $1/OSR^5$ ,  $\rightarrow$  eff. noise voltage is proportional  $1/OSR^{2.5}$ .

While the signal transfer function (STF) is flat, the noise transfer function (NTF) shapes the quantization noise according to  $1 / \text{integrator}^2 = \text{differentiator}^2$ . The transfer function of a time-continuous second order differentiator – modeled in the  $s$ -domain – is

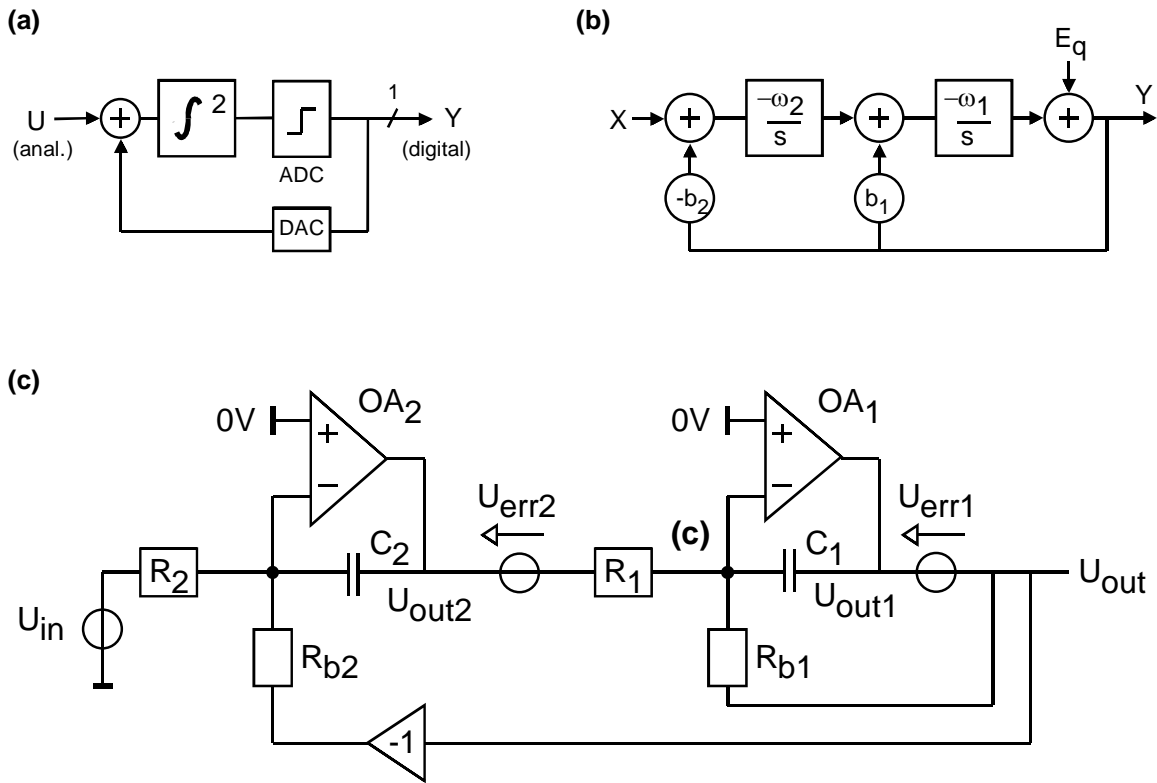
$$|H_{s,diff}(f)| = (f / f_T)^2.$$

with transit frequency  $f_T$ . Due to the clocked quantizer we have a time-discrete situation here. Consequently, the transfer function of a second order differentiator – modeled in the  $z$ -domain – becomes

$$|H_{z,diff}(F)| = C_2 \cdot \sin^2(\pi F),$$

with  $C_2$  constant,  $F=f/f_s$ . As  $\sin^2(x) \approx x^2$  for  $x \ll 1 \rightarrow 40\text{dB/dec}$  in the Bode diagram for  $F \ll 1$ .

Fig. 3.5-1 illustrates the 2<sup>nd</sup> order modulator to be used in this laboratory. Figure part (a) sketches the fundamental idea, (b) prepares it for modeling and (c) details the circuit.

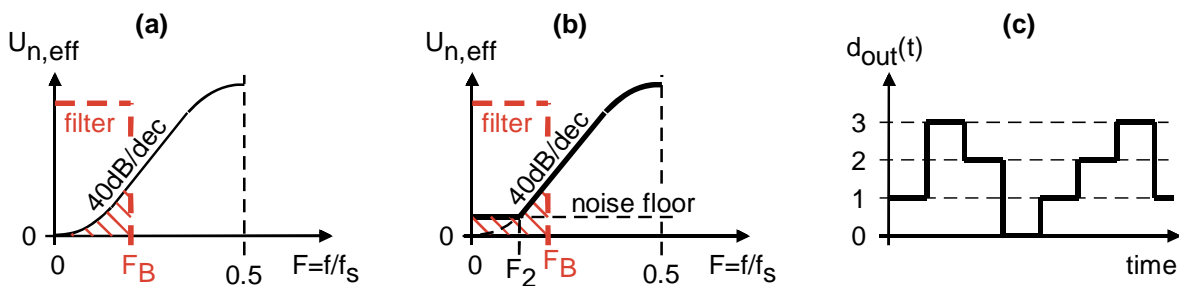


**Fig. 3.5-1:** (a) High-level schematics, (b) depicted for modeling, (c) realization with  $\omega_1=1/R_1C_1$ ,  $\omega_2=1/R_2C_2$ ,  $b_1=R_1/R_{b1}$ ,  $b_2=R_2/R_{b2}$  and error source  $U_{err1}$ , which will be replaced by the 2-level quantizer to obtain the  $\Delta\Sigma$  modulator.

**Fig. 3.5-2(a)** sketches how quantization noise within the baseband (i.e.  $f=0..f_B$  corresponding to  $F=0..F_B$  with  $F_B=f_B/f_s$ ) is reduced compared to a situation without noise shaping.

**Fig. 3.5-2 (b)** sketches the situation with the inevitable noise floor, because there are several noise sources other than the quantizer, too. The noise in the baseband is increased compared to Fig. part (a) as illustrated with the ideal lowpass.

**Fig. 3.5-2 (c)** illustrates overloading: A second order  $\Delta\Sigma$  modulator will always try to do some two-level jumps. If this is not possible, as for the 2-level quantizer, we say that the modulator is overloaded. Higher order modulators will try to perform N-level jumps (with N increasing with the modulator's order) and tend fail in convergence in case of overloading [1].

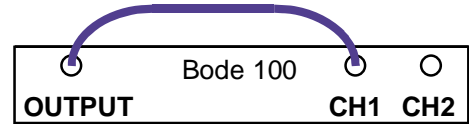
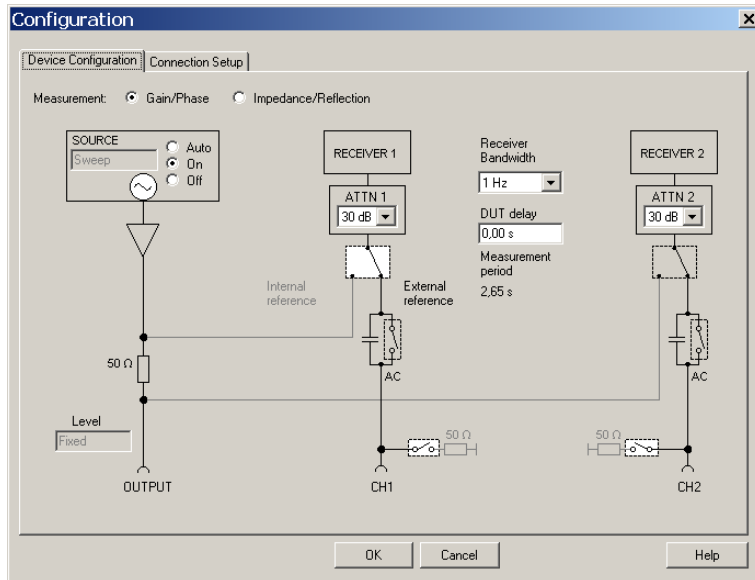


**Fig. 3.5-2:** (a) ideal situation, (b) situation with noise floor, (c) non-overloaded output behavior.

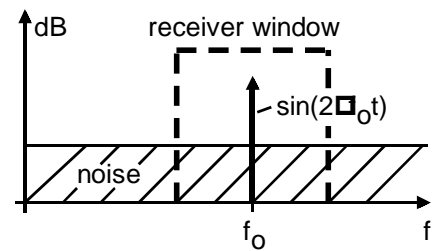
# 4 Laboratory Part

## 4.1 Knowing the Instruments

### 4.1.1 Starting the *BODE100* Vector Network Analyzer (10min, Σ 0:10h)



(b) External connection



(c) The receiver band width (RBW) decides about the amount of noise in your measurement.

(a) Conf. with *OUTPUT* not internally connected to *CH1*

**Fig. 4.1.1:** *BODE100* (a) setup and (b) external setup. (c) Getting noise acc. to RBW width.

Switch on power of *BODE100*, connect its USB cable to your PC and start the *Bode Analyzer Suite* (*BAS*) on PC. *BODE100* device No. (lower right corner of *BAS*): **CC156C**

In the *Configuration* menu set the switch in the middle of Fig. 4.1.1 to the right, so that *CH1* gets its voltage always from "External reference". Connect *CH1* externally to *OUTPUT* as shown in Fig. part (b). Part (c) illustrates: More *receiver bandwidth* measures more noise.

**Table 4.1.1:** Use following *BODE100* settings in the *Frequency Sweep* mode:

Start Frequency: <b>10Hz</b>	Stop Frequency <b>40 MHz</b>	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>201</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>30 Hz</b>

Connect *BODE100*'s *OUTPUT* to *CH2*, so that *OUTPUT*, *CH1* and *CH2* are connected and start a single sweep. *BODE100* computes  $U(CH2)/U(CH1)$ . Except from some resonant cable effects at high frequencies you should measure constantly 0dB amplitude and 0° phase.

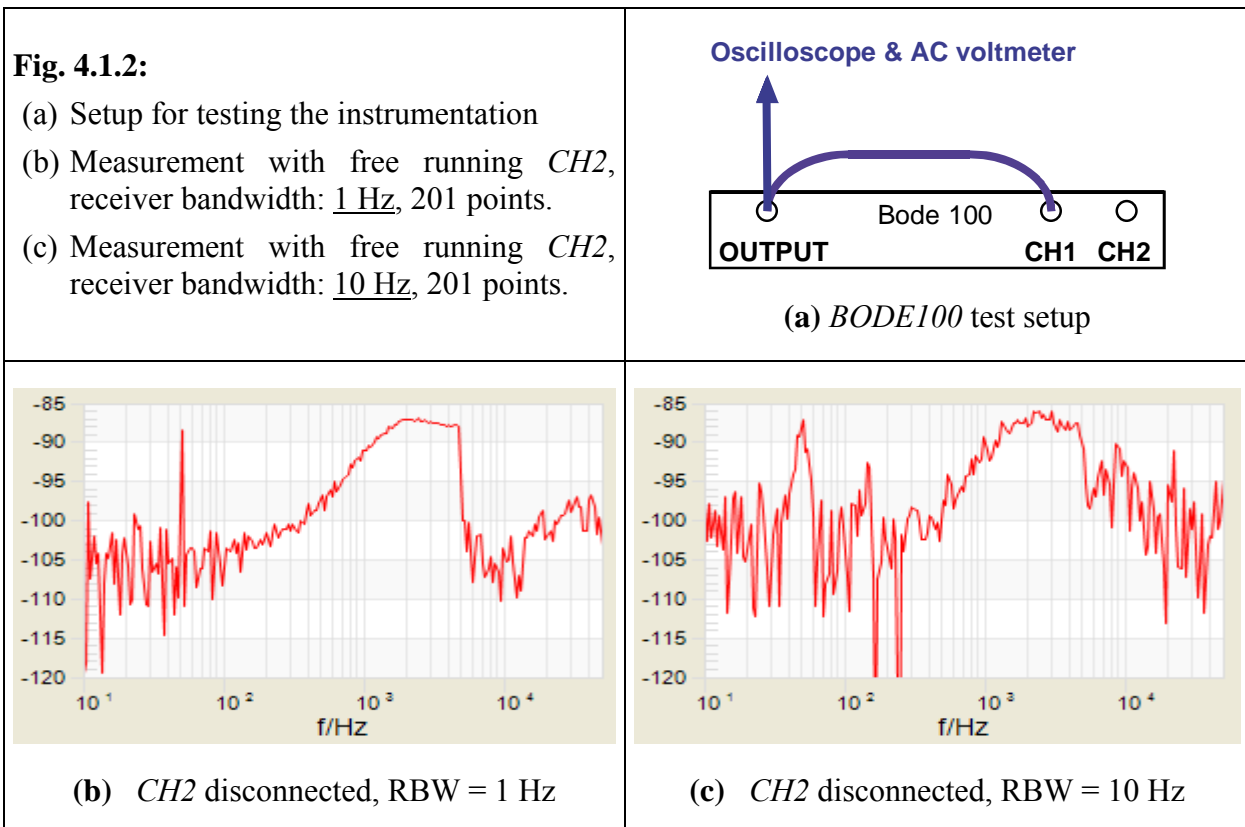
#### Frequency-Sweep:

Do you get constantly 0dB and 0° phase from 10Hz ... 1MHz? yes  
 (If not check your setup and / or ask your supervisor.)

Modify the Attenuator's damping: Strong change in measurement results? no

Set lower Receiver Bandwidths: Does this change the measurement speed? yes  
 (Measurement duration is approximately 3/RBW per point.)

### 4.1.2 Testing *BODE100*, Oscilloscope and AC Voltmeter (20min, $\Sigma$ 0:30h)



Note: *BODE100* will display the ratio of the voltages  $U(CH1)/U(CH2)$ .

*BODE100*'s *OUTPUT* keeps connected its to *CH1* while *CH2* is disconnected. Connect *BODE100*'s *OUTPUT* to your oscilloscope and AC voltmeter.

Set *BODE100* to *Gain/Phase* mode,

*OUTPUT*: Level=0dB, Source Frequency=1000 Hz, Attn *CH1*: 30dB, RBW 1Hz

Oscilloscope: Type: **Tektronix TDS2024B** , Peak-to-peak voltage? **1.3 V<sub>pp</sub>**

AC voltmeter: Type: **VoltCraft VC960** ,  $U_{\text{eff}}(f=1\text{KHz})?$  **445 mV**

Increase *BODE100*'s *OUTPUT* Level to the Overload limit of *CH1*  $\rightarrow$  *MaxLevel* = **17 dB** (*OUTPUT* Level is limited to  $\leq 13\text{dB}$ , but decreasing *CH1*'s Attenuation has same effect as increasing Level.)

*NoiseLevel* at 100Hz should be some  $-103\text{dB}$  as can be seen from Fig. 4.1.2(b). Compute the dynamic range *MaxLevel*-*NoiseLevel* at 100Hz (a) in dB, (b) as factor and (c) as **effective number of bits (ENOB)**. Use authors  $NoiseLevel|_{100\text{Hz}}$  of  $-103\text{dB}$  (or the slightly different value that you measured yourself).

Range at 100Hz =  $MaxLevel - NoiseLevel|_{100\text{Hz}} = \mathbf{. . 27\text{dB} . . - (-103\text{dB}) = \mathbf{. . 130 . . dB}$

This corresponds to a Factor =  $10^{6.5} = \mathbf{3162000}$ ,  $\rightarrow$  ENOB =  $\ln(\text{Factor})/\ln 2 = \mathbf{21.6}$  bits



**Spurious Free Dynamic Range (SFDR)** for 0...50 KHz, i.e.  $MaxLevel - \max\{NoiseLevel\}$ , in the range 0...50KHz. Use authors  $\max\{NoiseLevel\}|_{0...50KHz}$  of  $-87\text{dB}$  (or the slightly different value that you measured yourself).

$$SFDR = MaxLevel - \max\{NoiseLevel\}|_{0...50KHz} = \mathbf{..27dB..} - \mathbf{(-87dB)} = \mathbf{...114...dB}$$

This corresponds to a Factor =  $\mathbf{..501000...}$ ,  $\rightarrow ENOB = \ln(\text{Factor})/\ln 2 = \mathbf{18.9}$  bits.

### Check your AC Voltmeter

To check the valid frequency range of the voltmeter connect its input with *BODE100's OUTPUT* and adjust a measured, effective output voltage of  $1V_{\text{eff}}$  (using parameter *Level*). Don't waste time with being too accurate here!

Connect *BODE100's OUTPUT* to the AC voltmeter's input,

control measurement speed with receiver bandwidth, e.g.  $RBW=10\text{Hz}$ ,

$Level=7\text{dB} \rightarrow 1V_{\text{eff}}$  at  $50\text{Hz}$  (or adjust *Level* such that you measure  $1V_{\text{eff}}$  with your voltmeter)

*Freq. Sweep*:  $50\text{Hz}...1\text{MHz}$ .

The AC voltmeter shows ...

an error of  $+2\%$  at  $f_{+2\%} = \mathbf{....83....}$  KHz,

a maximum voltage of  $+\mathbf{...10...}$  % at frequency  $f_{\text{max}} = \mathbf{....290.....}$  KHz,

an error of  $-10\%$  at  $f_{-10\%} = \mathbf{...454...}$  KHz

### 4.1.3 Getting Started with the Board

(10min,  $\Sigma$  0:40h)

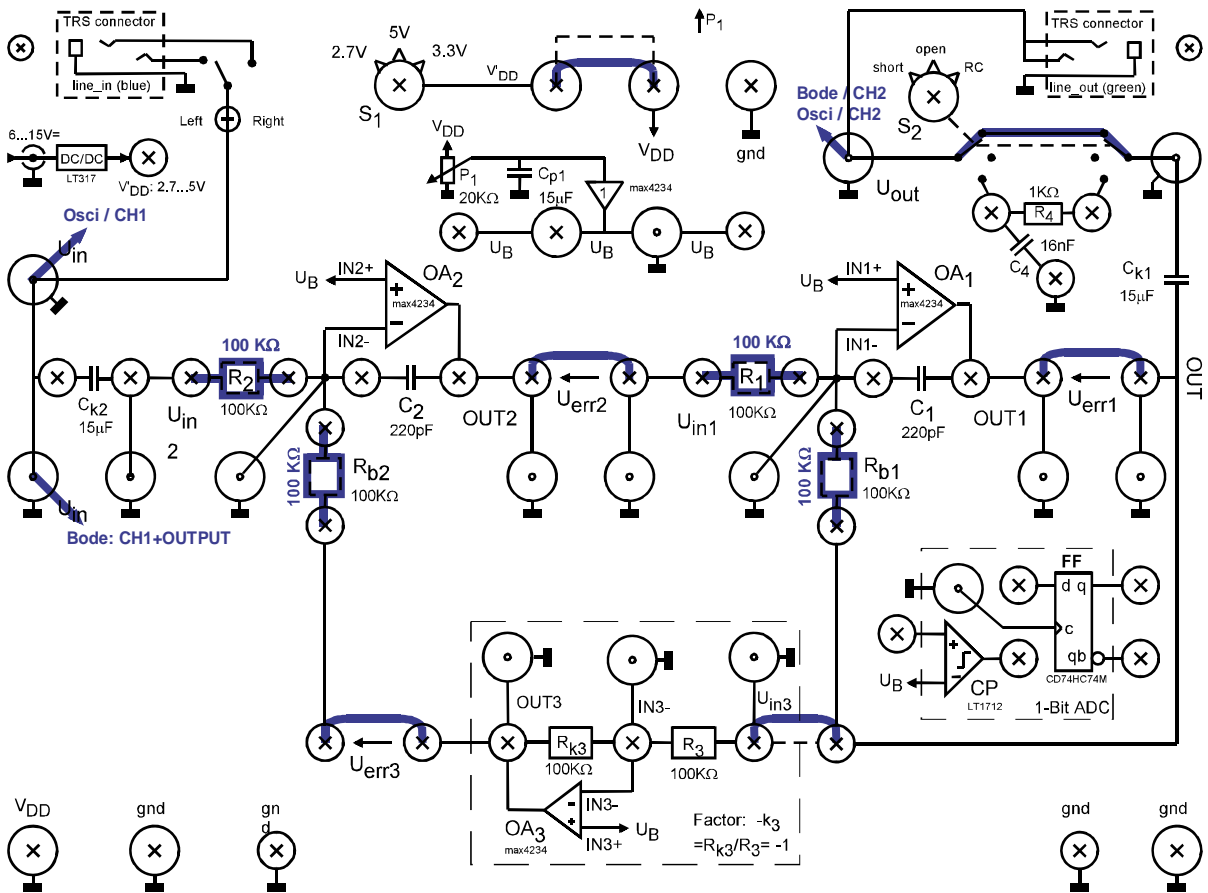


Fig. 4.1.3: Board configuration for first test.

**First of all:** disconnect clock generator, apply power plug, turn  $V_{DD}$  switch to 3.3V.

**Then** assemble the board as shown in the figure above.

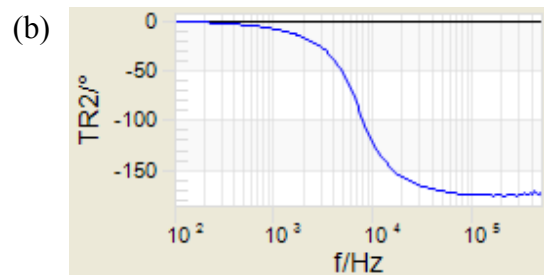
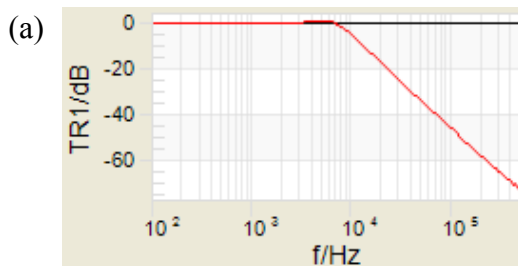
**Table 4.1.3:** Use following *BODE100* settings in the *Frequency Sweep* mode:

Start Frequency: <b>10 Hz</b>	Stop Frequency <b>500 KHz</b>	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>201</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>10 Hz</b>

Connect *BODE100*'s *OUTPUT+CH1* to Board's  $U_{in}$ , *CH2* to Board's *OUT*. Confirm diagram:

Screen Shot 4.1.3: Bode Diag.

$$\frac{U(OUT)}{U(Uin)}$$

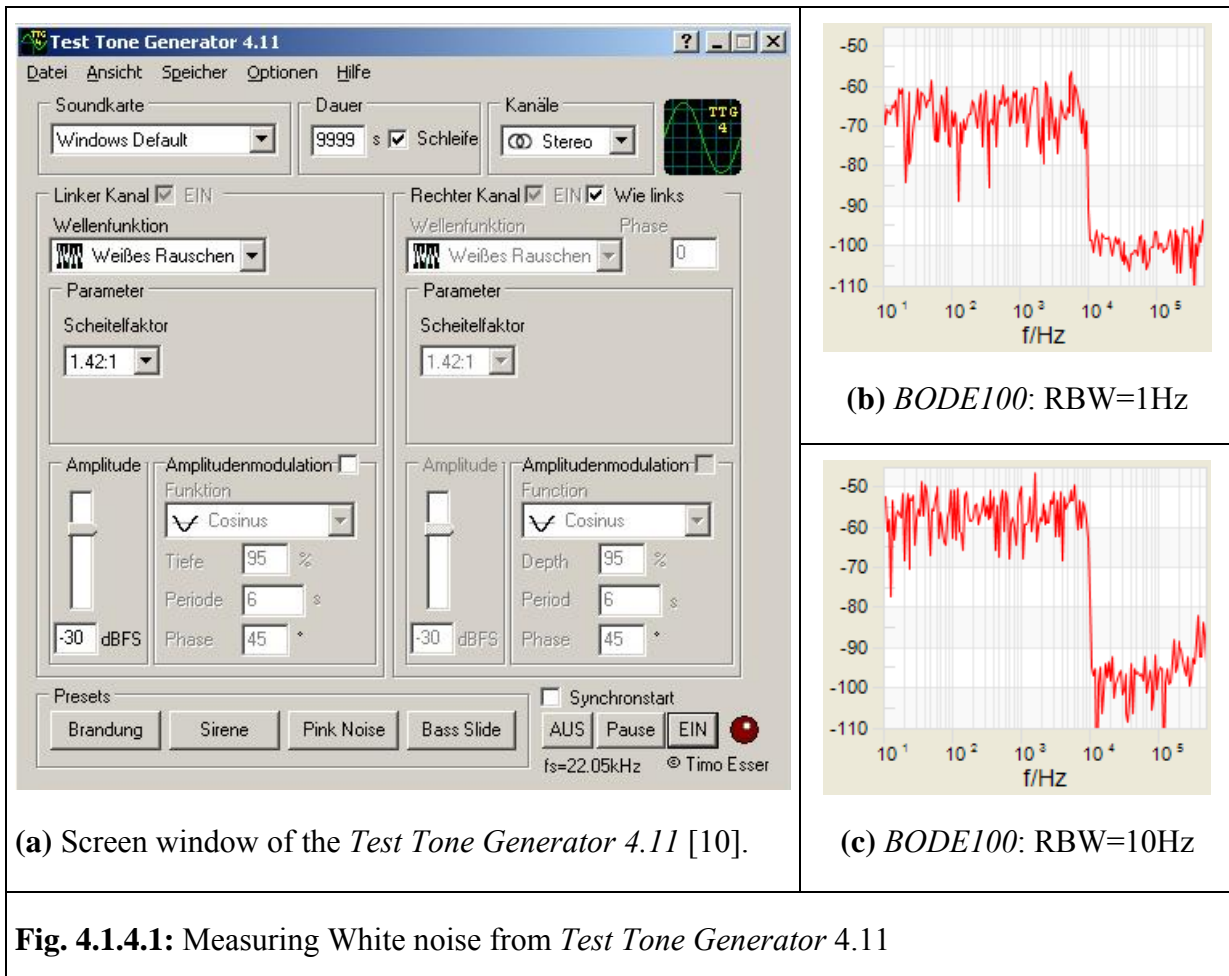


## 4.1.4 Using the PC's Soundcard as Signal Source

(20min,  $\Sigma$  1:00h)

### 4.1.4.1 White Noise Generation

(15min)



### Connect the PC's soundcard and the external speakers to the board

Disconnect *BODE100*'s *CH1* from the board's  $U_{in}$ . (Never connect two low-impedance sources!)

Connect PC-soundcard's "line out" (green) with the board's *TRS connector line\_in*.

Connect speakers to the board's *TRS connector line\_out*.

(*TRS* is an acronym for Tip, Ring, Sleeve, connected to left channel, right channel and ground, respectively)

**Start *Test Tone Generator 4.11*** [10], [11] as shown in Fig. 4.1.4.1(a) on your PC. Use settings as illustrated above. Set the soundcard of your PC to maximum output amplitude. Use your AC voltmeter to measure the effective output voltage:

**Measure  $u_{eff}$**  of the test-tone generator with settings above:

**22.6 mV**

**Table 4.1.4.1:** Use following *BODE100* settings in *Frequency Sweep* mode:

Start Frequency: <b>10 Hz</b>	Stop Frequency <b>500 KHz</b>	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>201</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>10 Hz</b>

**Question:** The test-tone generator can generate noise only up to ca. 10KHz. Why? ( $\rightarrow$  see  $f_s$ !)

**$f_s=22.05\text{KHz} \rightarrow \text{maximum frequency} = f_s/2 = 11.025 \text{ KHz}$**

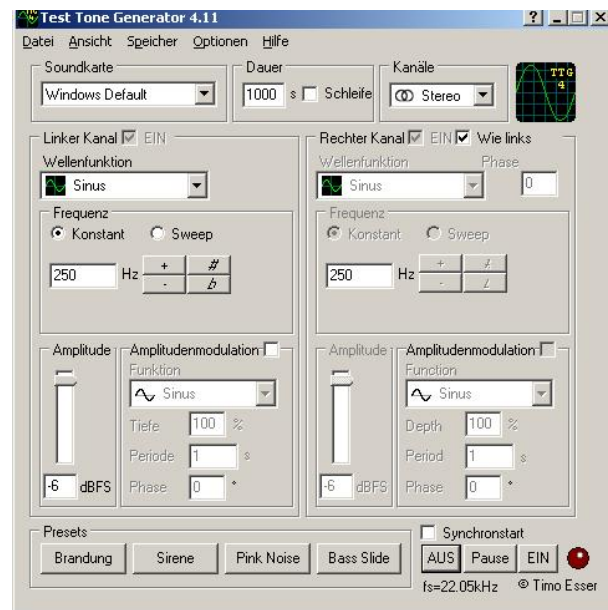
### 4.1.4.2 Sin-Wave Generation

(read only: 5min)

**Fig. 4.1.4.2:**

Settings of the *Test Tone Generator 4.11* [10] controlling the PC's soundcard: 220 Hz sinusoidal, 445mV<sub>eff</sub> (adjusted to 0 dB of *BODE100*)

Start Timo Esser's "*Test Tone Generator 4.11*" on your PC and adjust the settings window according to the figure right. Fig. 4.2.2-2(b) illustrates the spectrum measured for this settings with *BODE100*. The 50Hz peak might come from the 220V/50Hz power supply grid of the laboratory.



*BODE100*'s noise floor for the settings shown in Tab. 4.1.4.2 was checked with the measurement shown in Fig. 4.1.4.3(c) with an assembly according to figure part (a).

Then the sin-wave generator illustrated in Fig. 4.1.4.2 drove the PC's soundcard. The assembly according to Fig. 4.1.4.3(b) yielded the measurement shown in Fig. part (d).

**Fig. 4.1.4.3:**

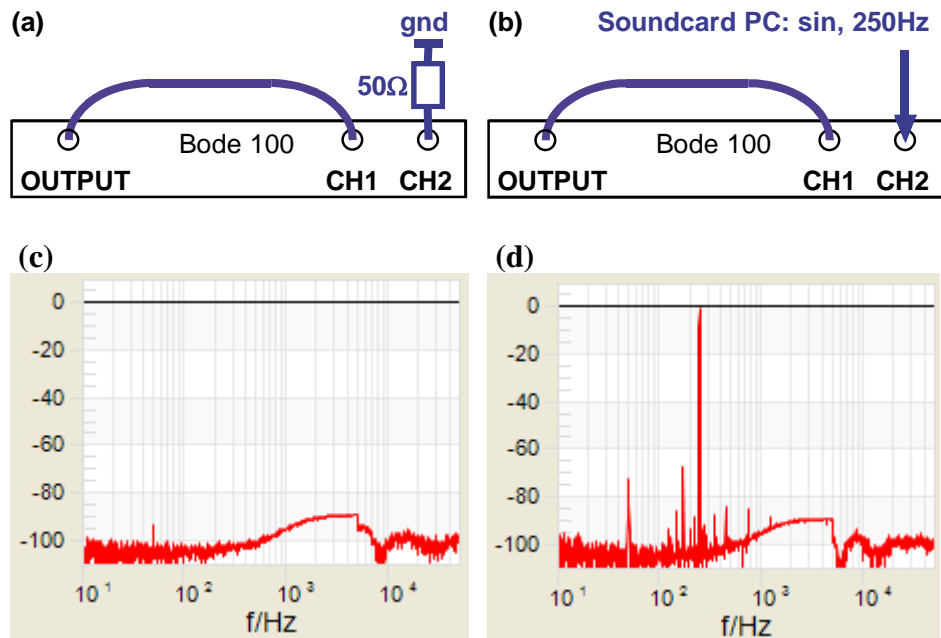
(a) Config. for (c)

(b) Config. for (d)

(c) *Bode100* measurement with config. (a).

(d) *Bode100* measurement with config. (b).

Each measurement acc. to Tab. 4.1.4.2 took some 70min.



**Table 4.1.4.2:** Use following *BODE100* settings in *Frequency Sweep* mode:

Start Frequency: <b>10 Hz</b>	Stop Frequency <b>50 KHz</b>	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>1601</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>1 Hz</b>

### 4.1.5 Comments on Using *Bode100* as Spectrum Analyzer (10min, $\Sigma$ 1:10h)

If you have spent more than 1 hour for chapter 4.1 of this laboratory, then you'd better continue with chapter 4.2 and read the following statements at home.

The *BODE100* is a network analyzer, not a spectrum analyzer. In the text above and below we use it in both modes: for network and spectrum analysis. The latter mode was chosen when *BODE100*'s *OUTPUT* is not used as input signal to the circuit under test. As *BODE100* is not intended for spectrum analysis some drawbacks have to be accepted in this mode:

1. For frequencies above 6KHz we have a superposition of the spectrum to be measured and a slightly shifted image of it. The shift varies from some 100Hz to some KHz depending on frequency and receiver bandwidth. This could result in indicated +3dB within the diagrams of this chapter. Fortunately, this effect is not very obvious in this lab.
2. Measurement results of spectral power densities like white noise depend on the receiver bandwidth (RBW), as will be shown and explained in more detail below.

As illustrated in Fig. 4.1.1(c) a sinusoidal signal corresponds to a Dirac function on the frequency axis so that the width of the receiver window has few impact on the measured signal power. However, white noise – being constant over the frequency axis – delivers 10 times more power within a 10 times wider bandwidth. A factor 10 in power corresponds to 10dB in the Bode diagram as visible for example in Figs. 4.1.4.1 (b) and (c).

Signals with different frequencies are always uncorrelated. Uncorrelated signals sum in power. Therefore, white-noise power grows linear with bandwidth and consequently the effective noise voltage grows with the square-root of the bandwidth. It is measured in  $V/\sqrt{Hz}$ . (Frequently **rtHz** is used as acronym for  $\sqrt{Hz}$ .)

In chapter 4.1.4.1 we measured  $u_{n,eff} = 22.6$  mV effective noise voltage generated by the sound card. It is equidistributed over a frequency range of 10 KHz. Consequently, the effective noise voltage caused by 1 Hz bandwidth is  $22.6 \text{ mV} / \sqrt{10000Hz} = 22.6 \mu\text{V} / \sqrt{Hz}$ .

For *Bode100* we measured an effective voltage of 445mV as 0dB. Consequently, the -60dB effective noise voltage measured in Fig. 4.1.4.1(b) with RBW=1Hz correspond to a factor 1/1000 resulting in  $445 \mu\text{V} / \sqrt{Hz}$ . This is about twice the value measured with the AC voltmeter and illustrates, that *BODE100* is not intended to perform accurate spectrum analysis. Nevertheless, it is useful to deliver valuable insight into noise shaping, the subject of this laboratory.

## 4.2 $\Delta\Sigma$ Modulation

### 4.2.1 Audiometric Test of 0<sup>th</sup>, 1<sup>st</sup>, 2<sup>nd</sup> Order Modulators. (20min, $\Sigma$ 1:30h)

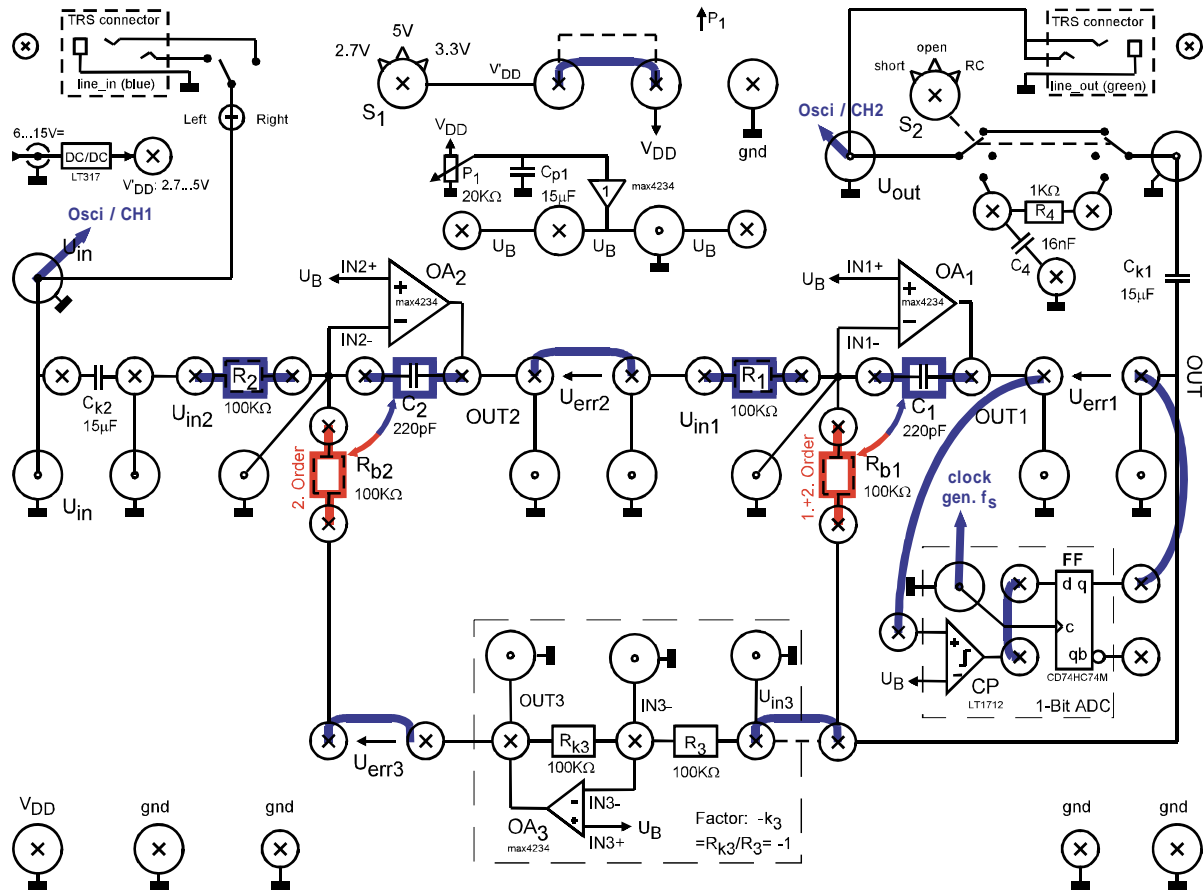


Fig. 4.2.1: Board configuration for modulators of 0<sup>th</sup>, 1<sup>st</sup> and 2<sup>nd</sup> order.

**Goal** of this subsection is to get an impression of the power of  $\Delta\Sigma$  modulation by simply **hearing** the differences in sound. In next subsections we will measure the differences.

**Do not destroy the flipflop's clock input!** Voltages  $< 0V$  and  $> V_{DD}$  can destroy the flipflop. **Before** connecting the clock generator to the FF's clock input make sure that

(a)  $V_{DD} \geq 2.7V$     **AND**    (b) the clock signal is within the range 0.1...2.7V

For clocking with Altera DE2 board and monitoring with DA2 board's DAC see chapter 5.

Assemble the circuit drawn in Fig. 4.2.1 with clock swing  $0 \dots V_{CC}$ .

#### First test:

Connect oscilloscope's *CHI* and *CH2* to the board's  $U_{in}$  and  $U_{out}$ , respectively.  $R_{b1}$ ,  $R_{b2}$  are in vertical positions (**red**). The board's *FF* should toggle now!

Connect PC's soundcard and speakers to the board

Disconnect *BODE100's CHI* from the board's  $U_{in}$ . (Never connect two low-impedance sources!)

Connect your PC-soundcard's "line out" (green) with the boards *TRS connector line\_in*.

Connect your speakers to the board's *TRS connector line\_out*.

Get sound from the internet (e.g. [www.youtube.com](http://www.youtube.com) or [www.antenne.de](http://www.antenne.de))  
Set switch  $S_2$  to "short". (It is located before the output to the speakers.)

### 0<sup>th</sup> order modulator

Set resistor  $R_{b1}$  horizontal (blue) and  $R_{b2}$  horizontal (blue) and rate the sound:

$f_s$	sound not identified										sound excellent	
	0	1	2	3	4	5	6	7	8	9		10
10 KHz	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
100 KHz	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1 MHz	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10 MHz	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

### 1<sup>st</sup> order modulator

Set resistor  $R_{b1}$  horizontal (blue) and  $R_{b2}$  vertical (red) and rate the sound:

$f_s$	sound not identified										sound excellent	
	0	1	2	3	4	5	6	7	8	9		10
10 KHz	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
100 KHz	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1 MHz	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10 MHz	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

### 2<sup>nd</sup> order modulator

Set resistor  $R_{b1}$  vertical (red) and  $R_{b2}$  vertical (red) and rate the sound:

$f_s$	sound not identified										sound excellent	
	0	1	2	3	4	5	6	7	8	9		10
10 KHz	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
100 KHz	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1 MHz	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10 MHz	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>

### Draw conclusions:

For the stand-alone quantizer ...

increasing  $OSR$  has  no  moderate  strong effect on sound quality

For a quantizer properly operated by a  $\Delta\Sigma$  modulator ...

increasing  $OSR$  has  no  moderate  strong effect on sound quality

increasing modulator order has  no  moderate  strong effect on sound quality

### Further options:

- You can switch  $S_2$  to include an RC lowpass into the signal path to the speakers.
- Or set  $S_2$  to "open" and bridge it with a 4<sup>th</sup> order lowpass,  $f_g=5\text{KHz}$ . (Don't short-circuit lowpass!)
- You may use a digital lowpass as  $\Delta\Sigma$  demodulator, available e.g. at HS.R [13].

### 4.2.2 Investigating Unshaped White Noise (0<sup>th</sup> Order) (20min, Σ 2:00h)

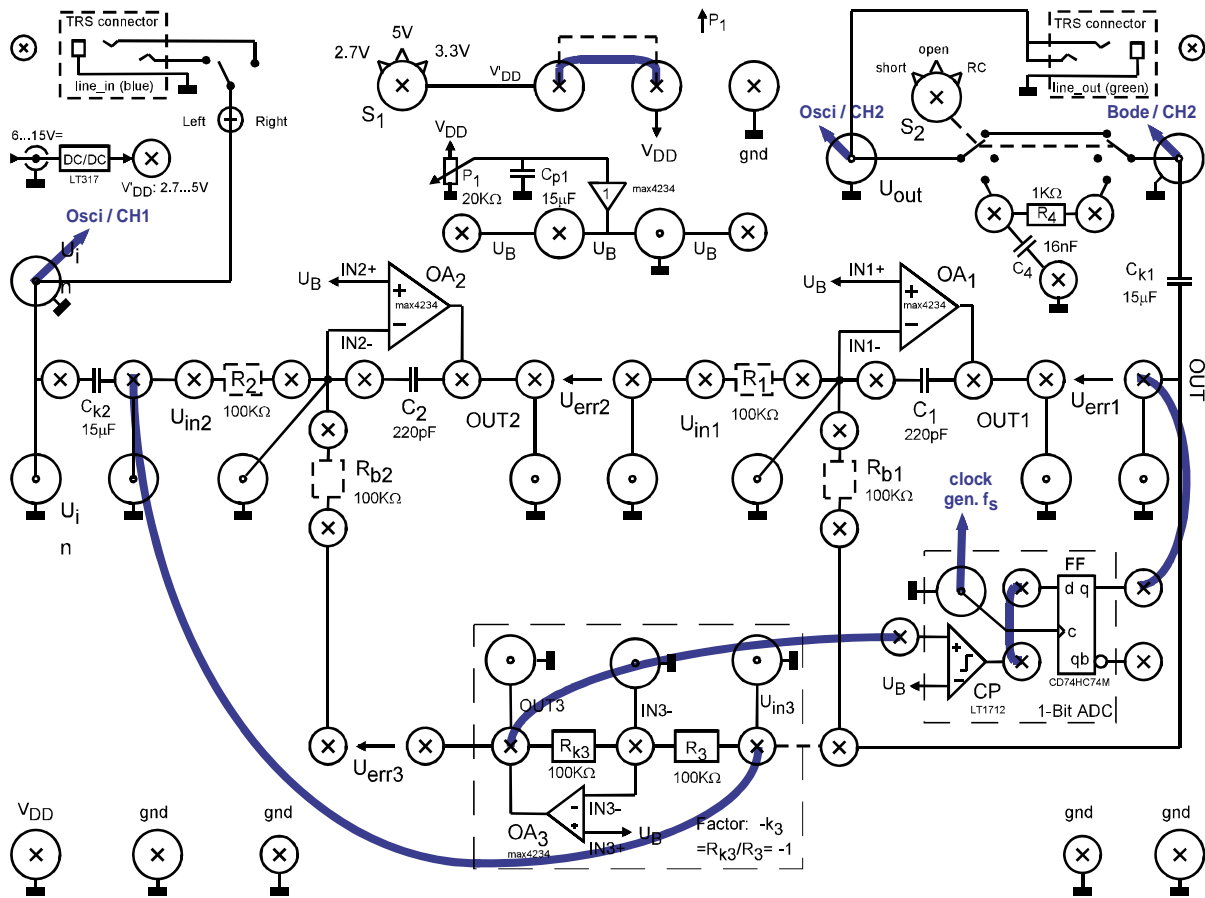


Fig. 4.2.2-1: Board configuration for unshaped noise (0<sup>th</sup> order).

**Goal:** Demonstrate that spectral power density of a random-bit stream is flat over frequency and proportional to  $\Delta^2/f_s$  in the frequency range  $0..f_s/2$ , while eff. voltage density is  $\approx \Delta/\sqrt{f_s}$ .

Quantization noise according to (2.10) is not easy to measure. We need to obtain the difference between unquantized and quantized signal, and often the first is analog and the latter digital. The way we go here is to generate a random bit-stream from a white-noise input signal  $u_{in}(t)$ , with  $|u_{in}(t)| \ll \Delta/2$ .

Choose *Test Tone Generator* settings according to Fig. 4.1.4.1(a) and feed the noise signal from your PC's sound card into the board's TRS connector *line\_in*. Assemble the board as illustrated in Fig. 4.2.2-1 above.

Table 4.2.2: Used *BODE100* settings in the *Frequency Sweep* mode:

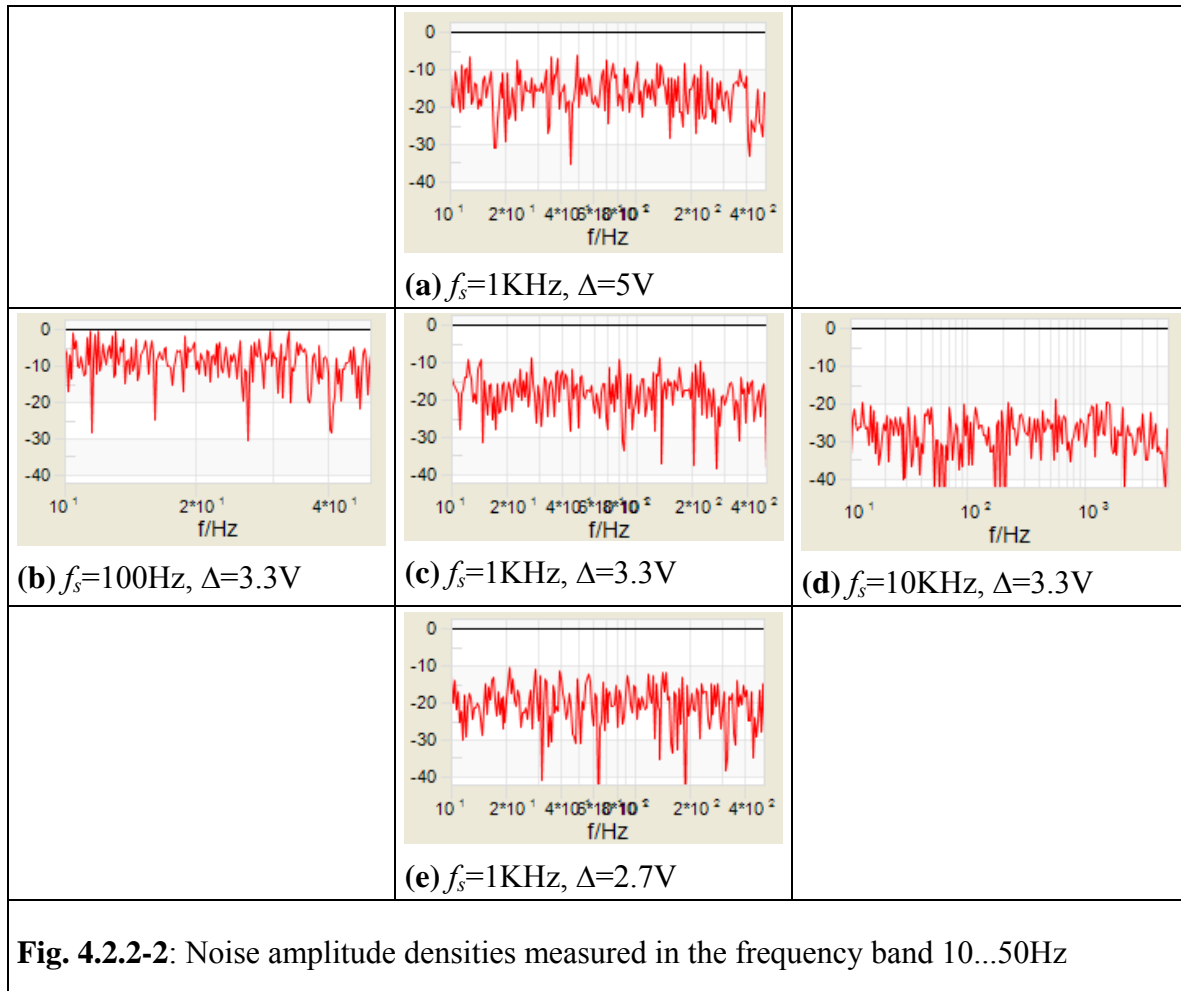
Start Frequency: <b>10 Hz</b>	Stop Frequency: $f_s/2$	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>201</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>1 Hz</b>

As *Bode100*'s measurement time per point is ca.  $T_{meas} \approx 3/RBW$  (4.2), the 201-point measurements shown in Fig. 4.2.2-2 take about 10 minutes each.



Settings:  $V_{DD}=2.7, 3.3, 5V$ , Sampling clock:  $f_s=100Hz, 1KHz, 100KHz$  with swing  $0...3.3V$ .

**Run the measurement for missing Fig. 4.2.2-2(c) while answering the questions below. Sketch the result in (C).** (101 points are enough. Begin with a fast test using  $RBW=10Hz$ .)



**Fill the following statements on the  $1/\sqrt{f_s}$  -relationship of spectral noise voltage density:**

From Fig. 4.2.2-2(b) to (d) the frequency range  $0...f_s/2$  is extended by a factor 100.

Consequently, the noise power density, which is  $\approx V^2/Hz$ , decreases by a factor **...100...**

and the noise voltage density measured in  $V/\sqrt{Hz}$  decreases by a factor **..  $\sqrt{100} = 10$ .**

Both corresponds to change on the logarithmic scale of **-20 dB**.

From Fig. 4.2.2-2(e) to (a)  $\Delta=V_{DD}$  increases by factor  $5 / 2.7 =$  **1.85** = **5.35 dB**

Consequently, the noise voltage density increases by factor  **$5/2.7=1.85$**  = **5.35 dB**

and the noise power density per  $Hz$  decreases by a factor **..  $1,85^2=3.43$ .** = **5.35 dB**

### 4.2.3 Investigating the 1<sup>st</sup> Order Delta-Sigma Modulator (20min, Σ 2:20h)

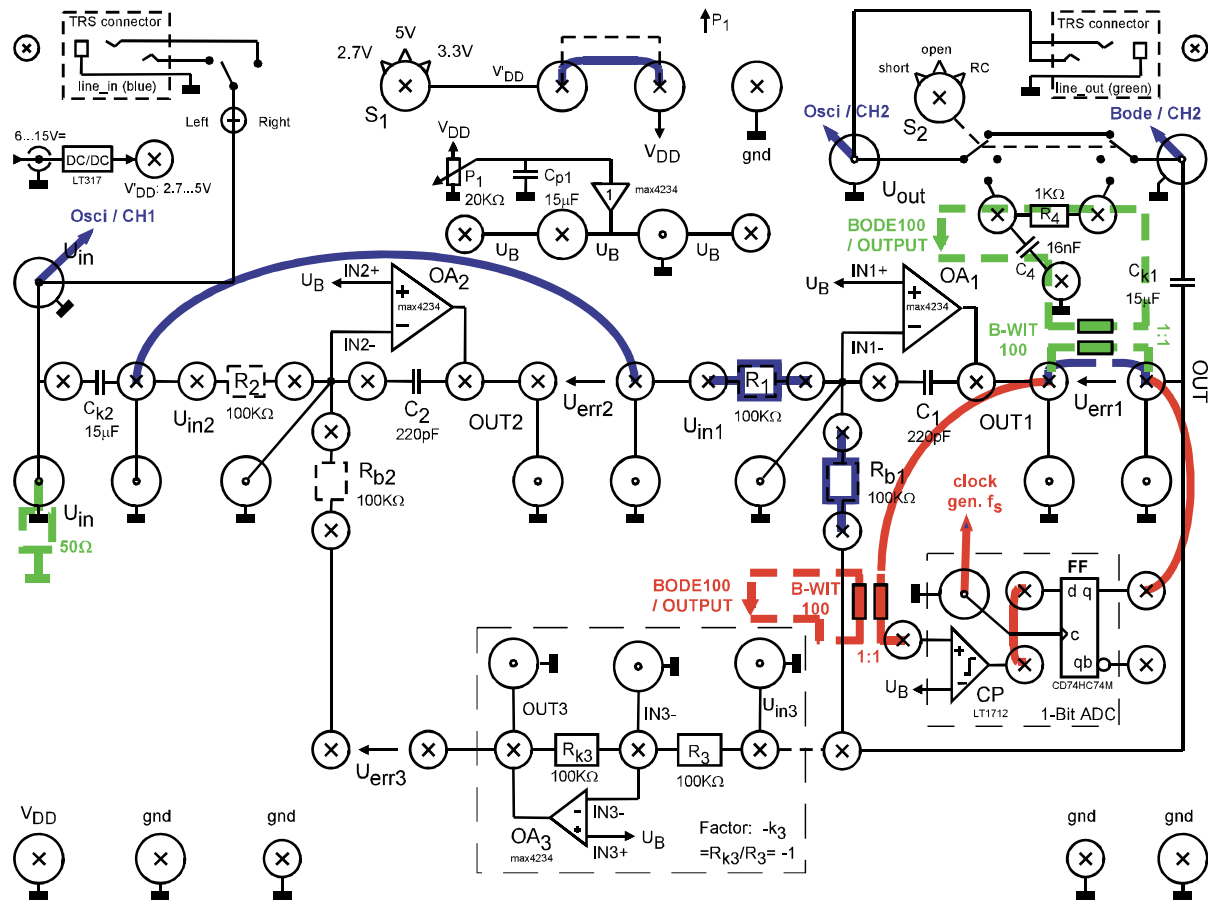


Fig. 4.2.3: Board configuration for 1<sup>st</sup> order modulator.

**Goal:** Run a 1<sup>st</sup> order  $\Delta\Sigma$  modulator, measure STF, NTF and noise shaping.

Table 4.2.3: Used following **BODE100** settings in **Frequency Sweep** mode:

Start Frequency: <b>10 Hz</b>	Stop Frequency <b>50 KHz</b>	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>201</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>10 Hz</b>

**Time-continuous transfer function (TF) measurements ( $V_{DD}=3.3V$ ):**

**STF, time-continuous:** Assemble the circuit as illustrated in Fig. 4.2.3 with blue solid and dashed lines, i.e.  $U_{err1}$  is a short circuit, comparator and flipflop not connected. Measure the signal transfer function (discussed detailed in [12]) with **BODE100** to confirm Fig. 4.2.5(a).

**NTF, time-continuous:** As indicated with dashed green lines in the figure above connect the Board's  $U_{in}$  to ground using a short circuit or 50Ω (or any resistor small compared to  $R_2=100K\Omega$ ). Remove the short-circuit over  $U_{err1}$  and generate  $U_{err1}$  by **BODE100's OUTPUT**, which is made independent from ground by the **B-WIT-100** isolating transformer. Keep **BODE100's CH1** connected to its **OUTPUT** and **CH2** to the board's  $U_{out}$ . Measure the noise transfer function (NTF) with **BODE100**. The result should comply with Fig. 4.2.5(c) [12].

### Time-Discrete TF measurements with 2-level quantizer:

Settings:  $V_{DD}=3.3V$ , Sampling clock:  $f_s=100KHz$  with swing  $0...3.3V$ .

Remove *B-WIT100* transformer.

Connect the flipflop's clock input to the clock generator (red line).

Connect the board's *OUT1* to *CP*'s  $IN^+$  (red line without *B-WIT 100*).

Connect and *CP*'s output to the *FF*'s *D*-input (red line)

Connect and *FF*'s output to the board's *OUT* (red line).

### STF, time-discrete:

Remove the  $50\Omega$  short circuit from  $U_{in}$  and connect  $U_{in}$  with *BODE100*'s *OUTPUT*.

Connect the board's  $U_{out}$  with *BODE100*'s *CH2*. (*CH1* remains connected to *OUTPUT*.)

Measure the *STF* with *BODE100* and sketch it in Fig. 4.2.5(b).

### NTF, time-discrete:

Connect the board's  $U_{in}$  with low impedance ( $\leq 50\Omega$ ) to ground.

Connect *B-WIT 100* transformer on the board between *OUT1* and the *CP*'s  $IN^+$  (red dashed).

Measure the *NTF* with *BODE100* and sketch the result in Fig. 4.2.5(d).

### Shaped noise measurements ( $V_{DD}=3.3V$ ):

Replace *B-WIT100* (red dashed) by a short.

Disconnect *BODE100*'s *OUTPUT* from  $U_{in}$ .

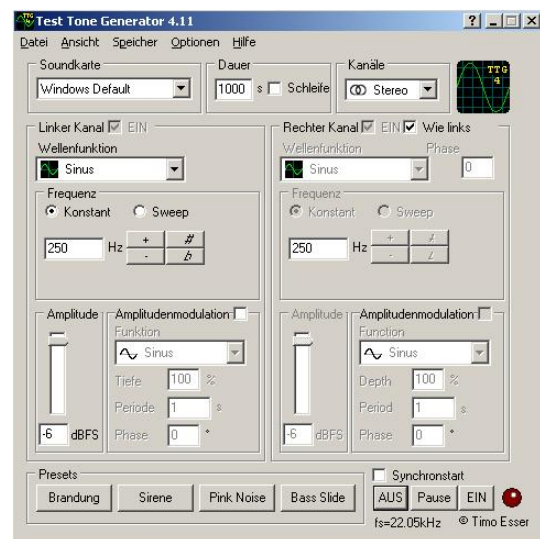
Connect  $U_{in}$  to your PC's soundcard.

Generate a 250Hz signal at *BODE100*'s 0dB-level with your soundcard (settings see right).

Perform a frequency sweep and confirm the measurement observed in Fig. 4.2.5(e). Does the measurement comply with your expectations of Fig. 3.4-2?

yes

.....



Test tome generator: 250Hz sinus

Complete the following sentences:

We observe (1) the **250Hz signal** as Dirac function, (2) the noise floor at

ca. **-90 dB** and (3) the  **$\sin(\pi F)$**  behavior for  $f > f_{NCF1}$  (noise corner frequency).

The  $f_{NCF1}$  is observed at about **20 Hz**.

The maximum slope of the noise shape for this 1<sup>st</sup> order modulator is **20 db/dec**

### 4.2.4 Investigating the 2<sup>nd</sup> Order Delta-Sigma Modulator (20min, Σ 2:40h)

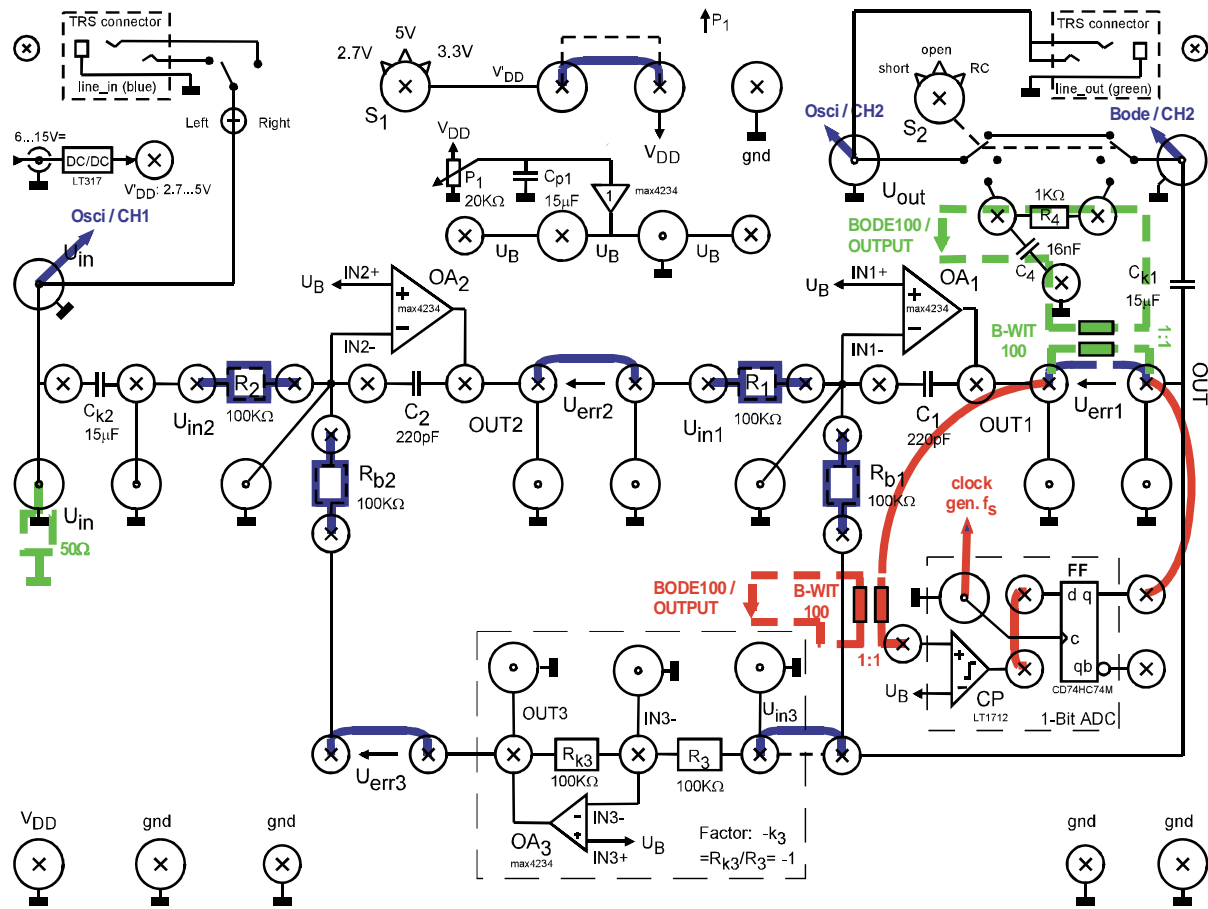


Fig. 4.2.4: Board configuration for 2<sup>nd</sup> order modulator.

**Goal:** Run a 2<sup>nd</sup> order ΔΣ modulator, measure STF, NTF and noise shaping.

Table 4.2.4: Used following **BODE100** settings in **Frequency Sweep** mode:

Start Frequency: <b>10 Hz</b>	Stop Frequency <b>50 KHz</b>	Sweep Mode: <b>Logarithmic</b>	Number of Points: <b>201</b>
Output Level: <b>0,00 dBm</b>	Attenuator CH1: <b>30 dB</b>	Attenuator CH2: <b>30 dB</b>	Receiver Bandwidth: <b>10 Hz</b>

**Time-continuous transfer function (TF) measurements (V<sub>DD</sub>=3.3V):**

**STF, time-continuous:** Assemble the circuit as illustrated in Fig. 4.2.4 with blue solid and dashed lines, i.e.  $U_{err1}$  is a short circuit, comparator and flipflop not connected. Measure the signal transfer function (discussed detailed in [12]) with **BODE100** to confirm Fig. 4.2.5(a).

**NTF, time-continuous:** As indicated with dashed green lines in the figure above connect the Board's  $U_{in}$  to ground using a short circuit or 50Ω (or any resistor small compared to  $R_2=100K\Omega$ ). Remove the short-circuit over  $U_{err1}$  and generate  $U_{err1}$  by **BODE100's OUTPUT**, which is made independent from ground by the **B-WIT-100** isolating transformer. Keep **BODE100's CH1** connected to its **OUTPUT** and **CH2** to the board's  $U_{out}$ . Measure the noise transfer function (NTF) with **BODE100**. The result should comply with Fig. 4.2.5(c) [12].

### Time-Discrete TF measurements with 2-level quantizer:

Settings:  $V_{DD}=3.3V$ , Sampling clock:  $f_s=100KHz$  with swing  $0...3.3V$ .

Remove *B-WIT100* transformer.

Connect the flipflop's clock input to the clock generator (red line).

Connect the board's *OUT1* to *CP*'s  $IN^+$  (red line without *B-WIT 100*).

Connect and *CP*'s output to the *FF*'s *D*-input (red line)

Connect and *FF*'s output to the board's *OUT* (red line).

### STF, time-discrete:

Remove the  $50\Omega$  short circuit from  $U_{in}$  and connect  $U_{in}$  with *BODE100*'s *OUTPUT*.

Connect the board's  $U_{out}$  with *BODE100*'s *CH2*. (*CH1* remains connected to *OUTPUT*.)

Measure the *STF* with *BODE100* and sketch it in Fig. 4.2.5(b).

### NTF, time-discrete:

Connect the board's  $U_{in}$  with low impedance ( $\leq 50\Omega$ ) to ground.

Connect *B-WIT 100* transformer on the board between *OUT1* and the *CP*'s  $IN^+$  (red dashed).

Measure the *NTF* with *BODE100* and sketch the result in Fig. 4.2.5(d).

### Shaped noise measurements ( $V_{DD}=3.3V$ ):

Replace *B-WIT100* (red dashed) by a short.

Disconnect *BODE100*'s *OUTPUT* from  $U_{in}$ .

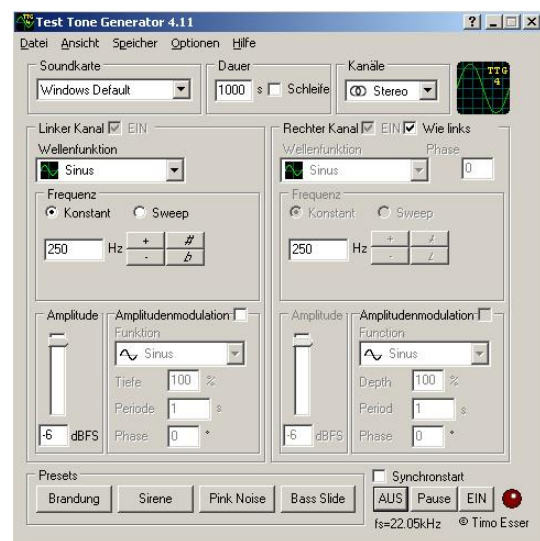
Connect  $U_{in}$  to your PC's soundcard.

Generate a 250Hz signal at *BODE100*'s 0dB-level with your soundcard (settings see right).

Perform a frequency sweep and confirm the measurement observed in Fig. 4.2.5(e). Does the measurement comply with your expectations of Fig. 3.5-2?

yes

.....



Test tome generator: 250Hz sinus

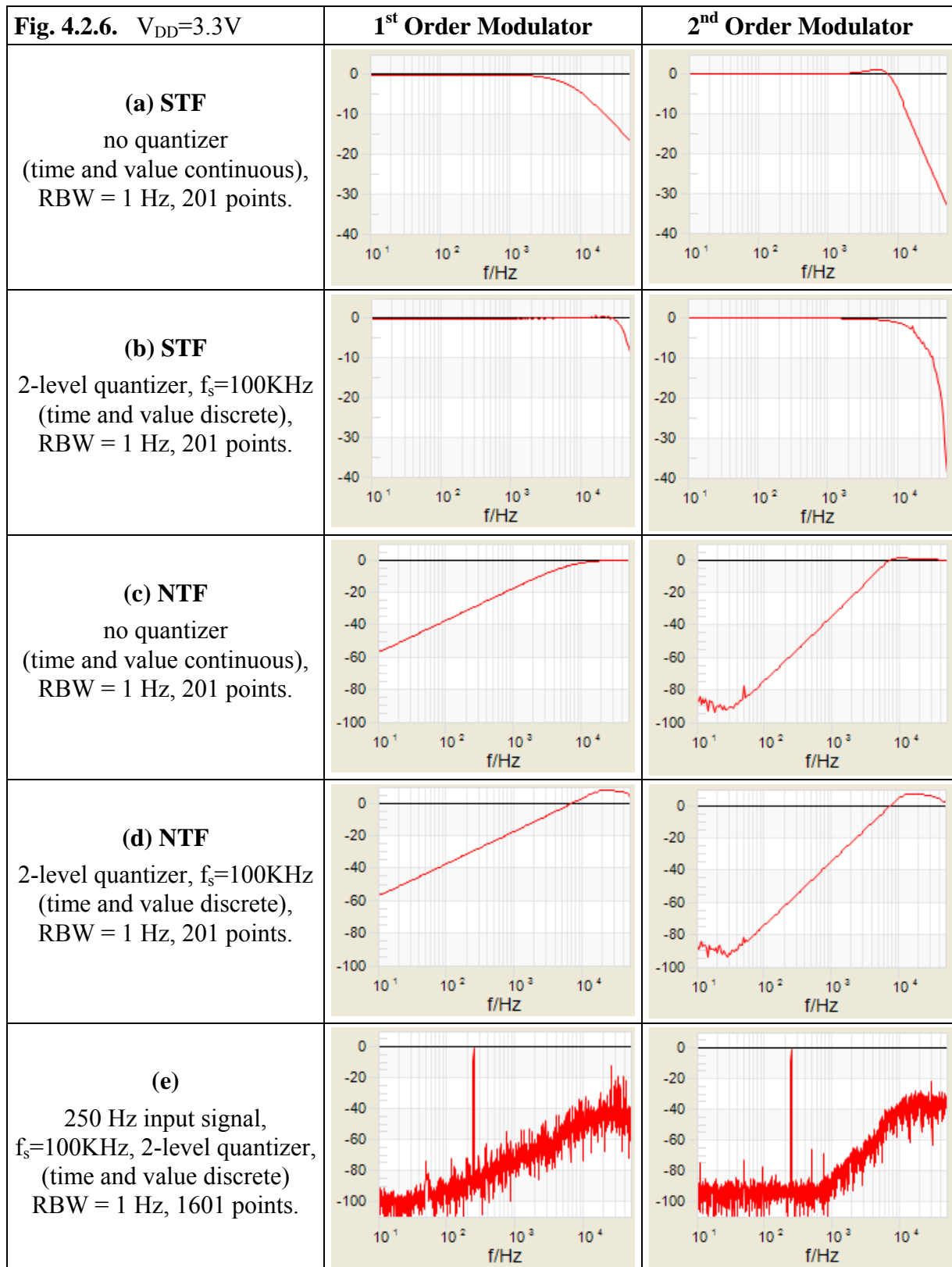
Complete the following sentences:

We observe (1) the **250Hz signal** as Dirac function, (2) the noise floor at ca. **-90 dB** and (3) the  **$\sin^2(\pi F)$**  behavior for  $f > f_{NCF2}$  (noise corner frequency).

The  $f_{NCF2}$  is observed at about **1000 Hz**.

The maximum slope of the noise shape for this 2<sup>nd</sup> order modulator is **40 db/dec**

### 4.2.5 Characteristics Summary of 1<sup>st</sup> and 2<sup>nd</sup> Order $\Delta\Sigma$ Modulators



### 4.3 Check your Knowledge

What is *OSR*? How does quantization-noise power in the baseband depend on the *OSR*? How does the effective quantization-noise voltage in the baseband depend on the *OSR*? Give both as functions of modulator order  $L$ . What 3 sections can you identify in the spectral noise voltage densities of Fig. 4.2.6(e)? Can you identify the noise corner frequencies? What function describes the rising section of the power-spectra for  $L=1$  and  $L=2$ ? What is the maximum slope of the rising spectra for  $L=1$  and  $L=2$  in dB/dec?

## 5 Conclusions

After presenting some fundamentals of signal processing in section 2, theory of  $\Delta\Sigma$  modulation was presented in section 3 and verified in section 4. Unshaped quantization noise behaves according to  $\Delta/\sqrt{f_s}$  with  $\Delta$  being the smallest step of the quantizer and  $f_s$  the sampling frequency.  $\Delta\Sigma$  modulated quantization noise behaves according to  $\Delta/OSR^{L+1/2}$ , where  $L$  is the order of the modulator and  $OSR=f_s/2f_B$  the oversampling ratio over baseband  $f_B$ . This formula includes for  $L=0$  the case of unshaped noise.

## 6 References

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- [4] M. Schubert, Courses at Regensburg University of Applied Sciences. Available: <http://homepages.fh-regensburg.de/~scm39115/homepage/education/courses/courses.htm>.
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- [8] M. Schubert, [4], Document → "BODE100 Quickstart for Lab1 & Lab2 at HS.R".
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- [12] M. Schubert, [4], Document → Laboratory Lab1: Analog Systems of 1<sup>st</sup> and 2<sup>nd</sup> Order.
- [13] M. Schubert, [4], Document → Digital Filter for  $\Delta\Sigma$  Demodulation (Lab 2 at HS.R).