Measurement of Skin Effects using the Bode 100

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Abstract: In electro engineering, metal conductors are used to transmit electric currents. An important thing to know is how the conductance of these conductors behave. Due to classical electrodynamics, we can classify conductivity in ohmic (purely resistive), inductive and capacitive effects. For constant currents, however, there is only an ohmic dependence. Since not all electric signals are constant, we also have to ask about the frequency dependent behavior which includes inductive and capacitive phenomena.

This article is mainly about a special ohmic behavior, the so called skin effect. There will be a brief introduction to the theory of skin effect as well as a short example how to measure skin effects using a cylindrical copper conductor. In order to proof, that we really measured skin effects and not some other inductive or capacitive phenomena, we added an appendix that explains the related theory.

1 Theory of the Skin Effects

First we want to mention some basics of electromagnetism. Let us define the current density J as the current per unit area. Now we consider a homogenous and isotopic conductor having a cylindrical form (diameter D, length L, resistivity ρ, magnetic permeability μ). We also neglect temperature influences.

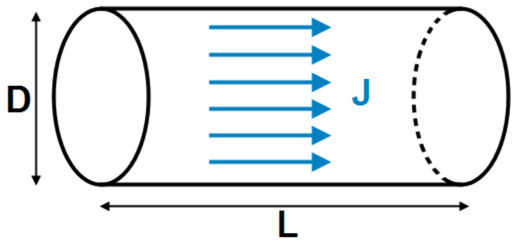

Figure 1: constant current

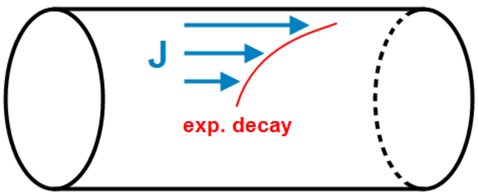

Figure 2: alternating current

If a constant current flows through our conductor, the current density J is homogenous (Figure 1) over the cross-section of our conductor and we can derive what usually is called Ohm's law.

$$U = R\, I \tag{1.1}$$

For alternating currents, the circumstances are different. Due to the magnetic field inside the conductor, eddy currents decrease the current density J, which, roughly spoken, "pushes the current" towards the boundary of the conductor. Namely J decays exponentially by a "skin depth" parameter δ. As a result most of the current flows in a small area near the surface of our conductor (Figure 2). This so called skin effect increases the effective resistance of the conductor at higher frequencies. The skin depth δ is given by

$$\delta = \sqrt{\frac{\rho}{\pi\ \mu\, f}} \tag{1.2}$$

If the skin depth δ is small compared to the conductor diameter D ($\delta \ll D$), a good approximation to the resulting effective resistance is

$$R_{skin} \approx \frac{L\, \rho}{\pi\, \delta\, D} \tag{1.3}$$

We can again use Ohm's law (1.1), but with the correct effective resistance of (1.3)

$$U = R_{skin}\, I \tag{1.4}$$

After this rough introduction to the physics of skin effect, we can continue and calculate the described effect for a real example.

To demonstrate the measurement of skin effects with the Vector Network Analyzer Bode 100, we will use a cylindrical copper conductor:

- Length L: 0.35 m
- Diameter D: 1.5 mm
- Resistivity ρ: 1.68E-08 Ωm
- Magnetic permeability µ: 1.256E-06 H/m

Having all needed parameters, we are now able to calculate the skin depth and the skin resistance for various frequencies.

Frequency f [Hz]	Skin Depth δ [m]	R_{skin} [Ohm]
1.00E+01	2.06E-02	δ<<D is not valid!
1.00E+02	6.52E-03	
1.00E+03	2.06E-03	
1.00E+04	6.52E-04	1.91E-03
1.00E+05	2.06E-04	6.05E-03
1.00E+06	6.52E-05	1.91E-02

Table 1: skin depth and effective resistance

Note: Only for frequencies higher than 10 kHz, our approximation is valid (the skin depth δ is more than 3 times smaller as D).

2 Measurement

2.1 Setup

Measuring the skin effect means to measure the change of impedance in a specified frequency range. Therefore we use the Impedance-Reflection mode of the Bode 100 and set it up as follows:

Measurement:	Frequency Sweep Mode
Start Frequency:	1 kHz
Stop Frequency:	1 MHz
Sweep Mode:	Logarithmic
Number of Points:	401 or more
Receiver Bandwidth:	100 Hz or lower
Level:	13 dBm

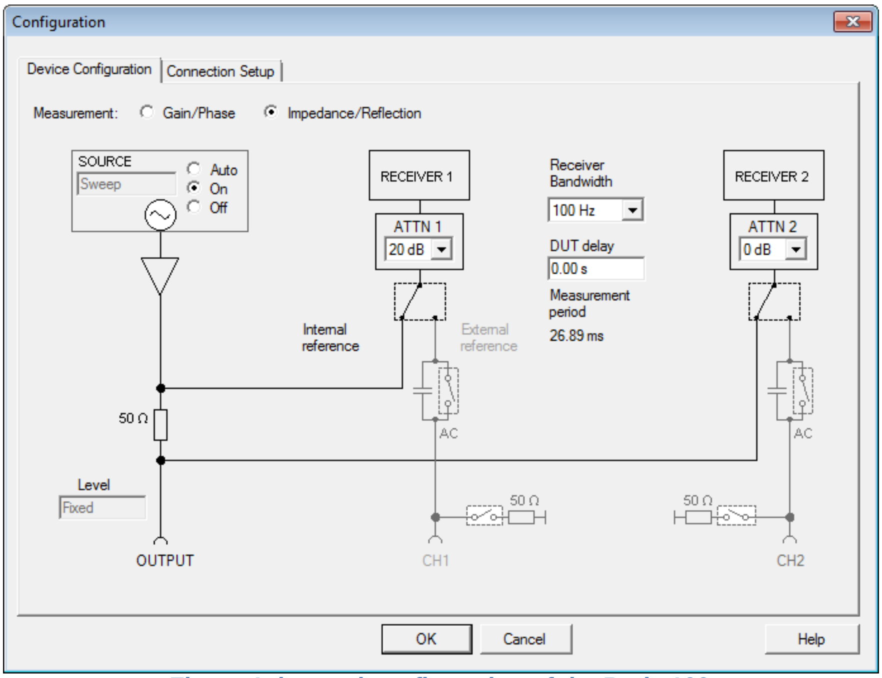

Figure 1: internal configuration of the Bode 100

The next step is to perform a user calibration in the impedance mode as shown below (open / short / load (50Ω)).

Figure 4: OPEN

Figure 5: SHORT

Figure 6: LOAD (50Ω)

Now it is time to introduce our DUT[1]:

Figure 7: DUT schematic

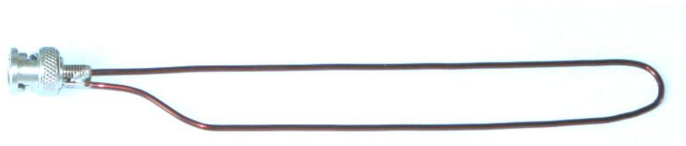
Figure 8: DUT assembling

Note: To minimize additional connection resistances, we assembled the BNC-Connector directly to our DUT. As shown in Figure 7 & 8, we have mounted the end of our conductor to the shielding of the connector. This avoids also possible ground loops.

[1] DUT Device Under Test (i.e. cylindrical copper conductor)

After calibrating the Bode 100 we can now connect our DUT.

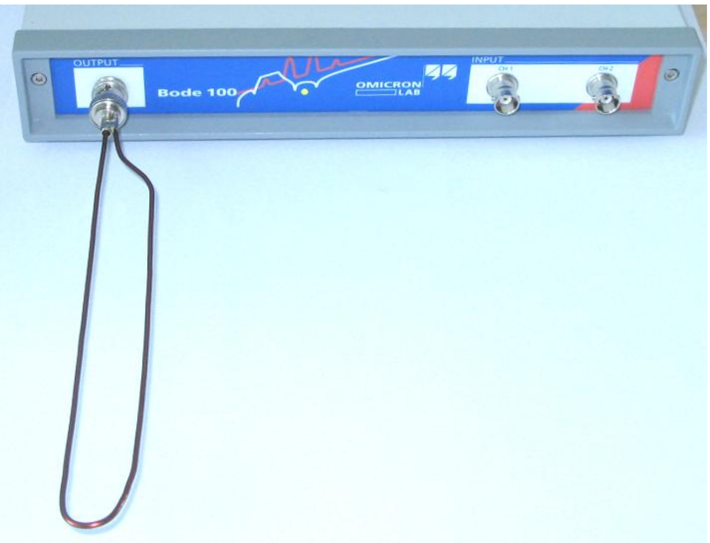

Figure 9: measurement setup

2.2 Measurement

Before starting the measurement, ensure that the trace settings are correctly adjusted (logarithmic Y-Scale, format real). Now everything is set up and we start our measurement by performing a single sweep.

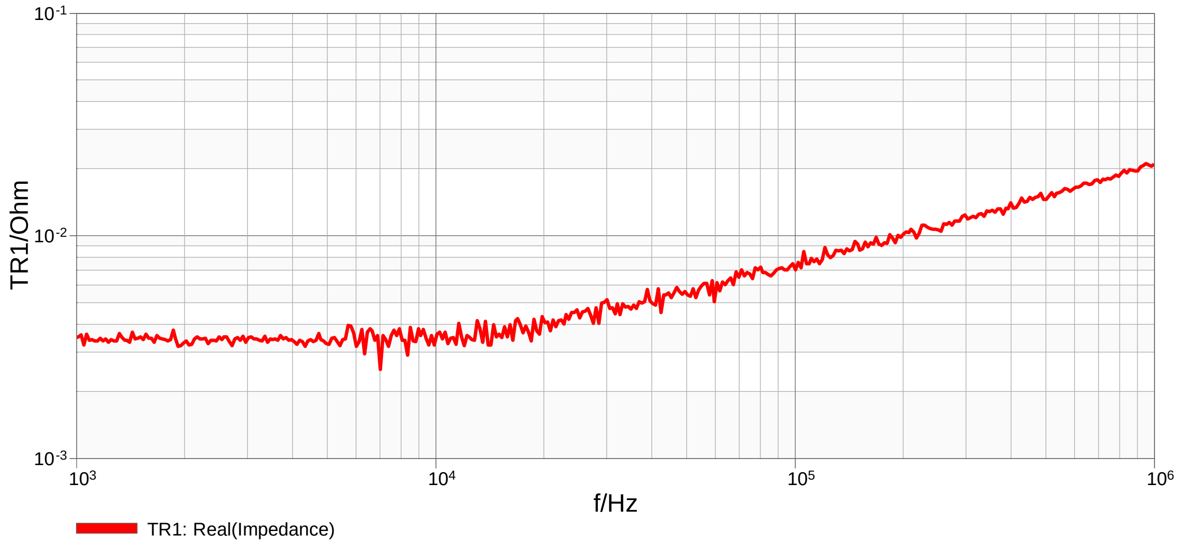

Figure 10: measured data trace

Note: Figure 10 shows the real part (ohmic) behavior of the impedance.

3 Conclusion

To compare the received data and the theoretical prediction, we export the trace data to a spreadsheet program and plot both, theoretical and measurement data.

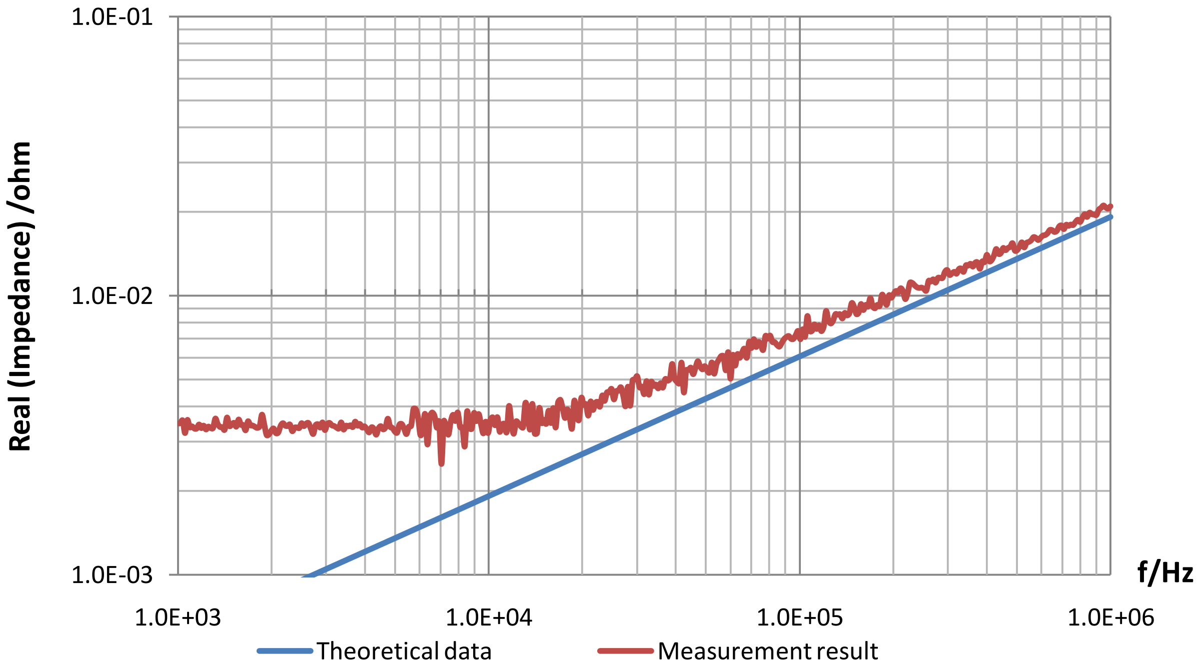

Figure 11: calculated and measured data

As we can see, for frequencies higher than 10 kHz, $\delta \ll D$ our approximation of the effective resistance is valid. The offset between theory and the measurement result may be caused due to the additional resistance of the used BNC-Connector.

Appendix: Inductance vs. ohmic Resistance

The physical parameters of the used conductor are:

- Length L: 0.35m
- Diameter D: 1.5mm
- Resistivity ρ: 1.68*10e-8 Ωm
- Magnetic permeability µ: 1.256*10e-6 H/m

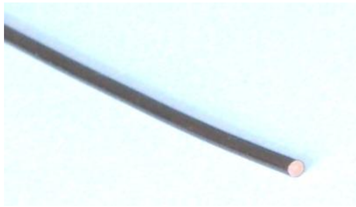

Figure A1

In general, any conductor at low frequencies can be described by an equivalent circuit, having two elementary electrical components as shown in Figure A2.

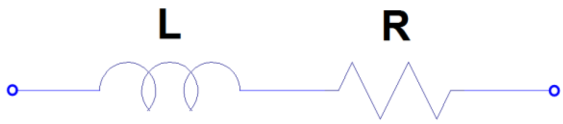

Figure A2: equivalent circuit

The equivalent circuit consists of:

- the series inductance L and
- the series resistance R which is the ohmic resistance of our conductor. Due to the skin effect R is frequency dependent

In Figure A3, it is shown where the input impedance of our DUT is measured.

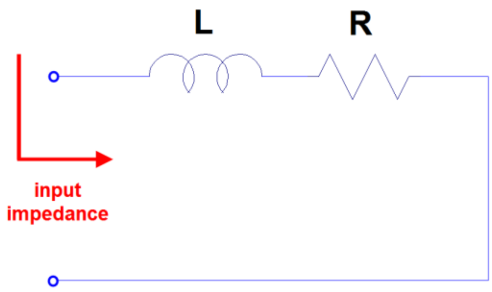

Figure A3: equivalent circuit measurement

The reactance X_L of an inductance L, where f is the frequency, is given by

$$X_L = 2\pi f L \tag{A.1}$$

A great advantage of the Bode 100 is that it can directly measure the impedance. The reactance X_L and the resistance R can then be displayed separately. Use now both traces to display the reactance[2] and the resistance[3] and perform a single sweep measurement.

[2] "Real" in the trace settings equals resistivity
[3] "Imag" in the trace settings equals reactance

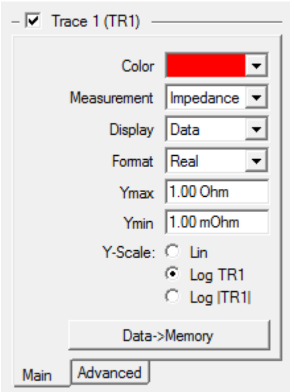

Figure A4: trace config.

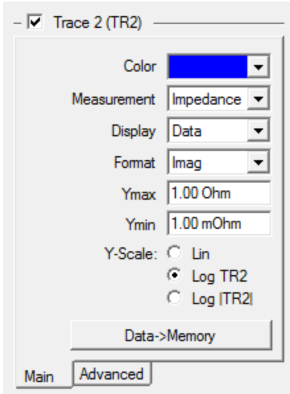

Figure A5: trace config.

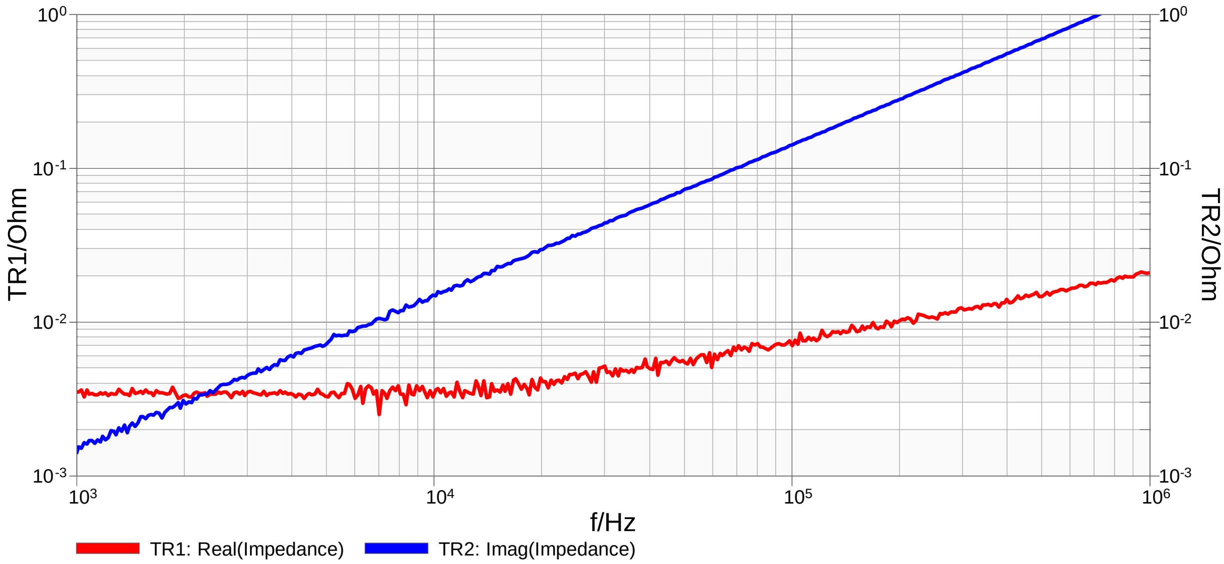

Figure A6: inductance and resistance measurement

How to interpret this result?
From the formulas (3.2) and (3.3) we can obtain that for $\delta \ll D$ (for frequencies > 10 kHz) the resistance is proportional to the square root of the frequency

$$R \sim \sqrt{f} \tag{A.2}$$

On the other hand we have just mentioned (see A.1) that the reactance is proportional to the frequency

$$X_L \sim f \tag{A.3}$$

Since we have a logarithmic trace axis, we can derive

$$\log(\mathrm{R}) \sim \log\left(\sqrt{\mathrm{f}}\right) = \frac{1}{2}\log(\mathrm{f}) \sim \frac{1}{2}\log(X_L) \tag{A.4}$$

Using the cursor feature of Bode 100, one can easily verify correlation (A.4) (i.e. the gradient of resistance is only half the gradient of reactance in logarithmic scale). This is a very quick verification that the slope measured for trace 1 (red) in Figure A5 is not caused by inductive or capacitive effects.