

This is for

- Teachers in electronics
- Electronic Lab exercises

Lesson time: 3-4 Hours

Prerequisite: Understanding how crystals are described in an equivalent circuit.

High school				Technical School				University			
1 st	2 nd	3 rd	4 th	1 st	2 nd	3 rd	4 th	1 st	2 nd	3 rd	4 th

Lesson Plan: Measurement of the equivalent circuit of a quartz crystal

1 Introduction

1.1 Why should students listen to your quartz lesson?

Where is quartz used in modern electronics?

What do you think is the reason quartz is used so often?

Quartz is used in electronic circuits to get a high accuracy time standard. Accuracy you need in clocks, computers, controllers, game boys, toys and many more.

1.2 Target: After this lesson

You can answer following questions:

- What is the serial resonance?
- What is the parallel resonance?
- What is the anti-resonance?
- How does the Q range of quartz compare to a RLC resonance circuit?
- Why is the quartz frequency 32.768KHz so popular?

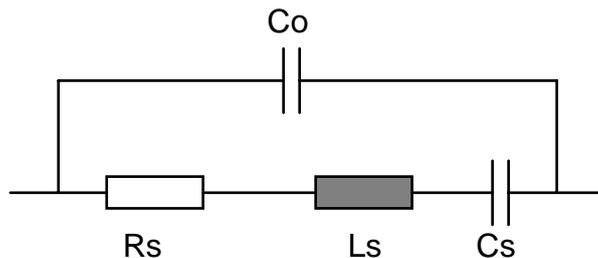
1.3 Your tasks after this lesson

After the lesson we want that you go home and check on the Internet what you can find about quartz. You will find some circuits & schemes so that you can see how they are implemented.

2 Main Part

2.1 Theory

An equivalent description of a quartz crystal is given by the following circuit. It is valid in the region of a single Series-Parallel resonance combination. As can be shown these combinations occur at odd multiples (odd overtones of the crystal) of the fundamental Series resonant frequency.



With known values of L_s , C_s , C_o and R_s we obtain the following equations:

$$\text{Series resonant frequency: } f_s = \frac{1}{2 \cdot \pi \cdot \sqrt{L_s \cdot C_s}}$$

$$\text{Parallel resonant frequency: } f_p = f_s \cdot \sqrt{1 + C_s / C_o} \approx f_s \cdot \left(1 + \frac{C_s}{2 \cdot C_o}\right)$$

$$\text{Quality factor at } f_s: \quad Q = \frac{2 \cdot \pi \cdot f_s \cdot L_s}{R_s} = \frac{1}{2 \cdot \pi \cdot f_s \cdot C_s \cdot R_s}$$

As we don't know these values we have to solve the equations for L_s , C_s first, because we can measure C_o , f_s and f_p with the Bode 100.

$$C_s = \left(\frac{f_p}{f_s} - 1 \right) \cdot 2 \cdot C_o$$

$$L_s = \frac{1}{4\pi^2 f_s^2 C_s}$$

Instead of using the above relationships it is possible to calculate the equivalent circuit out of a different set of measurements using the following equations :

$$\text{Loaded } Q \text{ at } f_s: \quad Q_L = \frac{2 \cdot \pi \cdot f_s \cdot L_s}{2 \cdot R + R_s} \quad \text{with } R = 50\Omega \text{ for a } 50\Omega \text{ measurement setup}$$

$$\text{Quality factor at } f_s: \quad Q = Q_L \cdot \left(1 + \frac{2 \cdot R}{R_s}\right)$$

$$L_s = \frac{Q \cdot R_s}{2 \cdot \pi \cdot f_s} \quad C_s = \frac{1}{Q \cdot 2 \cdot \pi \cdot f_s \cdot R_s}$$

2.2 Measurement of C_0 of the crystal:

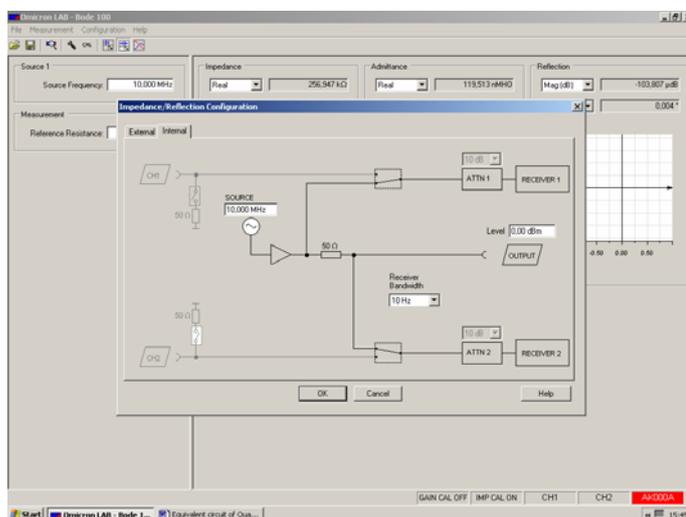
The Device Under Test is the crystal mounted at the Test PCB delivered with the Bode 100. We want to determine the equivalent circuit values at the fundamental frequency:

Our quartz crystal has a nominal Series resonant frequency of 12MHz. The first task is to measure the parallel Capacitance C_0 . How can we do this?

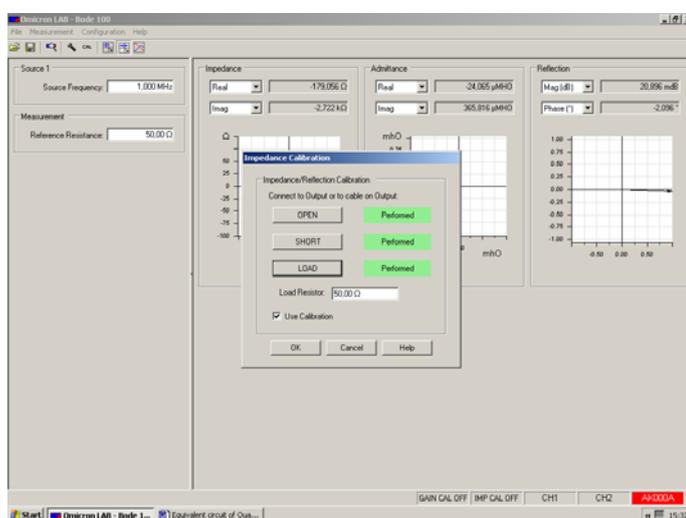
We use Bode 100 to measure the impedance of the crystal at a frequency that is well apart from the Series-Parallel resonant frequencies. For the 12MHz crystal 10MHz will be a good value. The result will be a nearly pure reactance of capacitive type. Before we start the measurement we have to set the measurement frequency, the measurement level and we have to perform an impedance calibration at the end of the connection cable used for the measurement.

Start the Impedance/Reflection measurement, open the internal configuration window and make following settings:

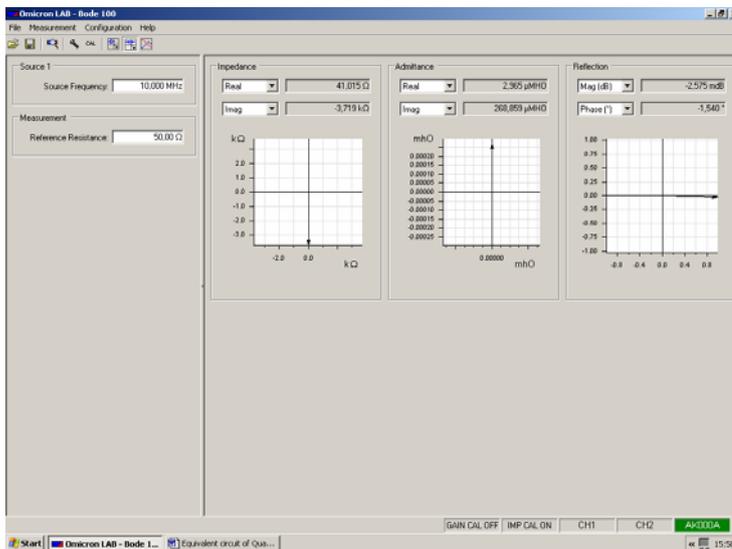
Source frequency: 10MHz
 Receiver Bandwidth: 10Hz (minimum bandwidth to avoid measurement errors caused by noise)
 Level: 0dBm (preset value)



Open the Calibration window and perform the impedance calibration by measuring “Open”, “Short” and “Load” as described in the Bode 100 user manual.



Now we are ready to measure the Impedance of the crystal. Connect the measurement cable to the “IN” BNC connector and the “Short” to the “OUT” BNC connector of the Quartz filter on the Test PCB. By using right click and pressing “Optimize” in each diagram you will get the following display:



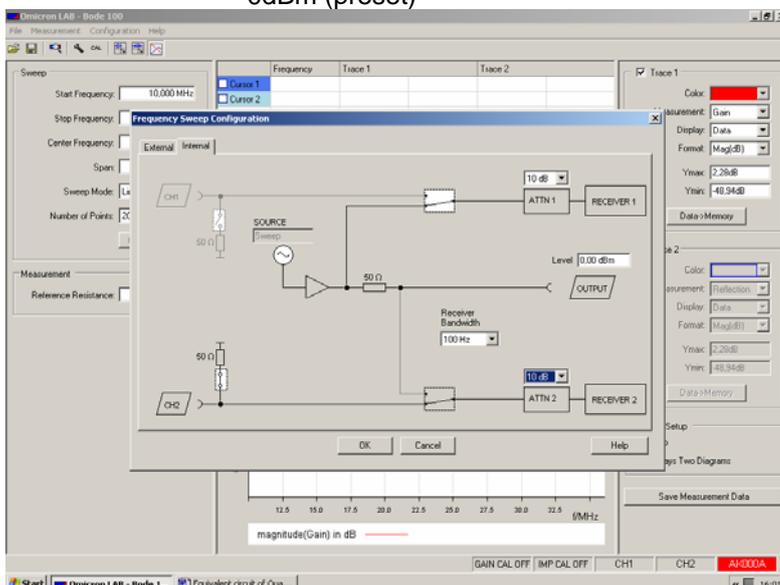
The readout for the Impedance is: $Z = 41.015 - j 3719 \text{ Ohm}$ at 10MHz and with $X_c = -3719 \text{ Ohm}$ we get:

$$C_o = \frac{1}{2\pi f |X_c|}$$

$$C_o = 4.28 \text{ pF}$$

2.3 Measurement of the loaded Q and Rs of the crystal:

Now we have to measure the loaded Q at f_s and the Series resistance of the crystal. For this purpose we start the frequency sweep Gain measurement, connect the crystal to OUTPUT and CH2 by means of 50 Ohm cables, open the internal configuration window and make following settings:
 CH1 and CH2 attenuator: 10dB
 Receiver Bandwidth: 100Hz
 CH2 impedance: 50 Ohm Internal reference (preset)
 Level: 0dBm (preset)



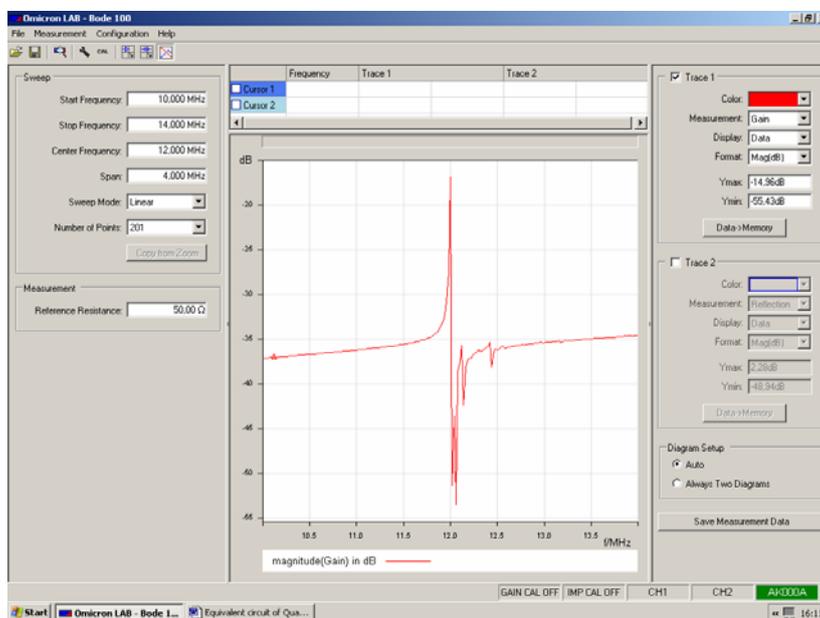
Leave the configuration window by pressing OK and make following settings in the Frequency sweep window:

Start 10MHz

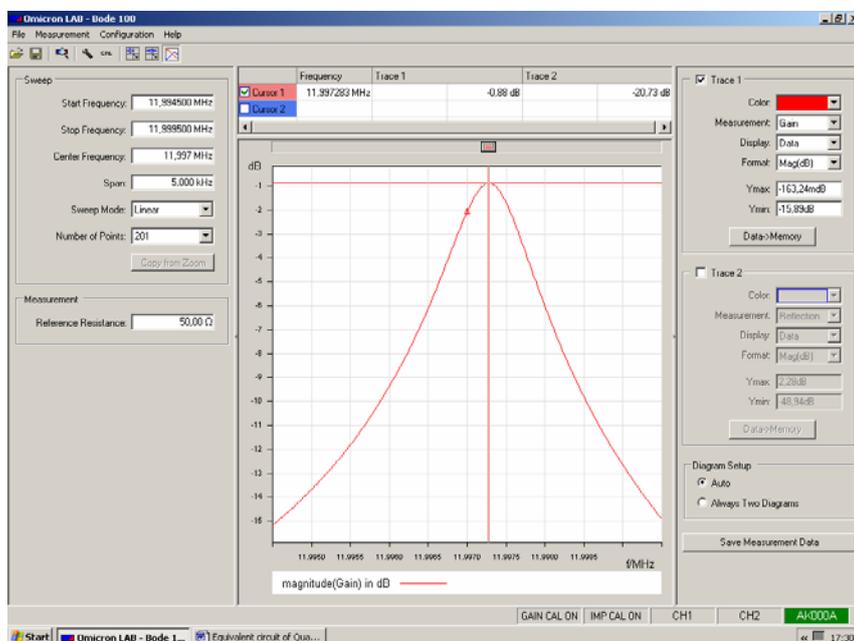
Stop 14MHz

Switch off Trace 2

use preset settings for Trace 1 and Optimize to produce the following trace:



As we can see, the frequency span is too high, so we have to zoom in by right click and selecting Zoom mode. Zooming in and applying Copy from Zoom will produce a closer look to the interesting range around f_s .



Using the cursor 1 and cursor 2 we get $f_s = 11.997339\text{MHz}$ and $B_{3\text{dB}} = 978\text{Hz}$ and $Q_L = 12267$

Finally we have to measure the Series Resistance R_S to calculate Q of the crystal. One possibility is to use the measured attenuation at the Series resonant frequency, the other way is to measure the impedance of the crystal at the Series resonant frequency.

The readout of cursor 1 gives us an attenuation of 0.88dB. To calculate R_S we have to use the following equation:

$$R_S = 2 \cdot R \cdot \left(10^{\frac{a}{20}} - 1 \right)$$

$$R_S = 100 \cdot \left(10^{\frac{0.88}{20}} - 1 \right) = 10.66 \text{ Ohm}$$

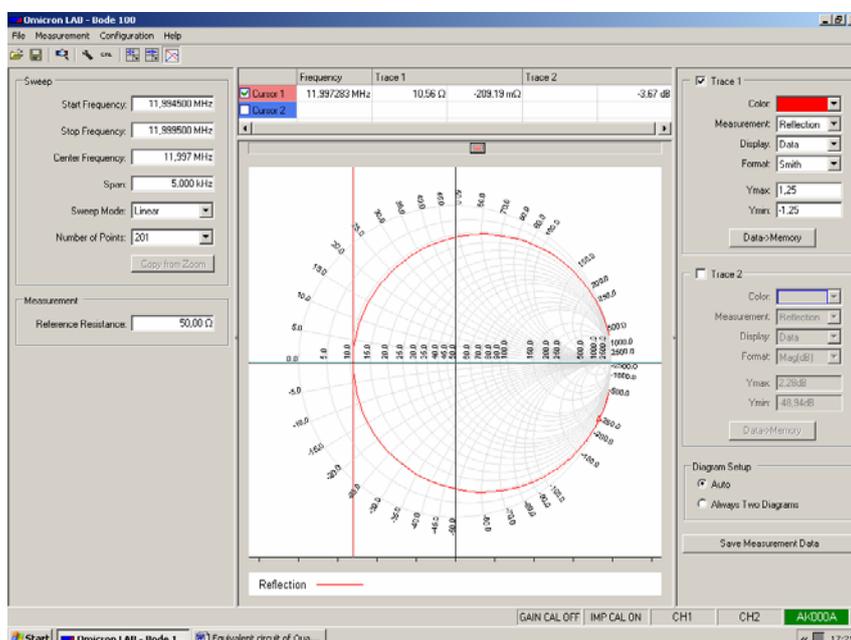
a ... attenuation in dB

R ... Source resistance, Receiver resistance : 50 Ohm both for Bode 100

To check our result obtained from the gain measurement, we perform a Reflection measurement. Select Reflection in the Measurement field and Smith chart in the Format field for Trace 1.

Now we have to perform the impedance calibration in the sweep mode.

After calibration connect the measurement cable to the "IN" BNC connector and the "Short" to the "OUT" BNC connector of the Quartz filter on the Test PCB. The following trace should be displayed now:



Place cursor 1 to the left most point of the trace, which corresponds to the Series Resonant frequency.

The value of R_S is 10.56 Ohm with this measurement.

The values of R_S are almost the same for both methods.

2.4 Calculation of the complete equivalent circuit of the crystal

First we have to calculate the quality factor of the crystal itself (unloaded Q).

With $Q_L = 12267$, $R_s = 10.6 \text{ Ohm}$ and $R = 50 \text{ Ohm}$ we get $Q = 127993$

$$\begin{aligned} C_0 &= 4.28 \text{ pF} \\ R_s &= 10.6 \text{ Ohm} \\ L_s &\approx 18 \text{ mH} \\ C_s &\approx 9.8 \text{ fF} \\ Q &\approx 128000 \end{aligned}$$

2.5 Some numbers and facts of crystal oscillators in different applications

- Microcontroller applications, frequency range from 1MHz up to 50MHz, accuracy uncritical about $\pm 200\text{ppm}$, frequency deviation at 10MHz less than 2kHz
- Transmitter applications in cable head ends, often 10MHz oscillators as reference frequency for phase locked loops, these oscillators are temperature compensated to obtain better frequency stability, accuracy better than $\pm 10\text{ppm}$, frequency deviation at 10MHz less than 100Hz this means that the frequency deviation of a 10MHz oscillator is lower than 100Hz !
- Standard transmitter applications, often 10MHz oscillators as reference frequency for phase locked loops, these oscillators heated to temperatures in the range between 60°C and 90°C to obtain better frequency stability, accuracy better than $\pm 1\text{ppm}$ this means that the frequency deviation of a 10MHz oscillator is lower than 10Hz !
- Oscillators used as frequency standard, often combined with references as GPS or other frequency standards, these crystal oscillators are heated in double control loops and reach frequency deviations of less than 10mHz/day
- Oscillators in clocks – the frequency is 32.768Hz. This frequency is divided by 2^{15} (binary divider) and we get one pulse per second. If we suppose that the oscillator has an accuracy of 10ppm our clock will deviate 5 minutes in one year.
- Some pictures of crystals

