Unveiling the Mathematics Behind NISM:
A New Path to Power Integrity/Power Supply Stability

Steven M. Sandler - Picotest

15th OMICRON Lab Power Analysis & Design Symposium 2026 (VIRTUAL)
April 29, 2026

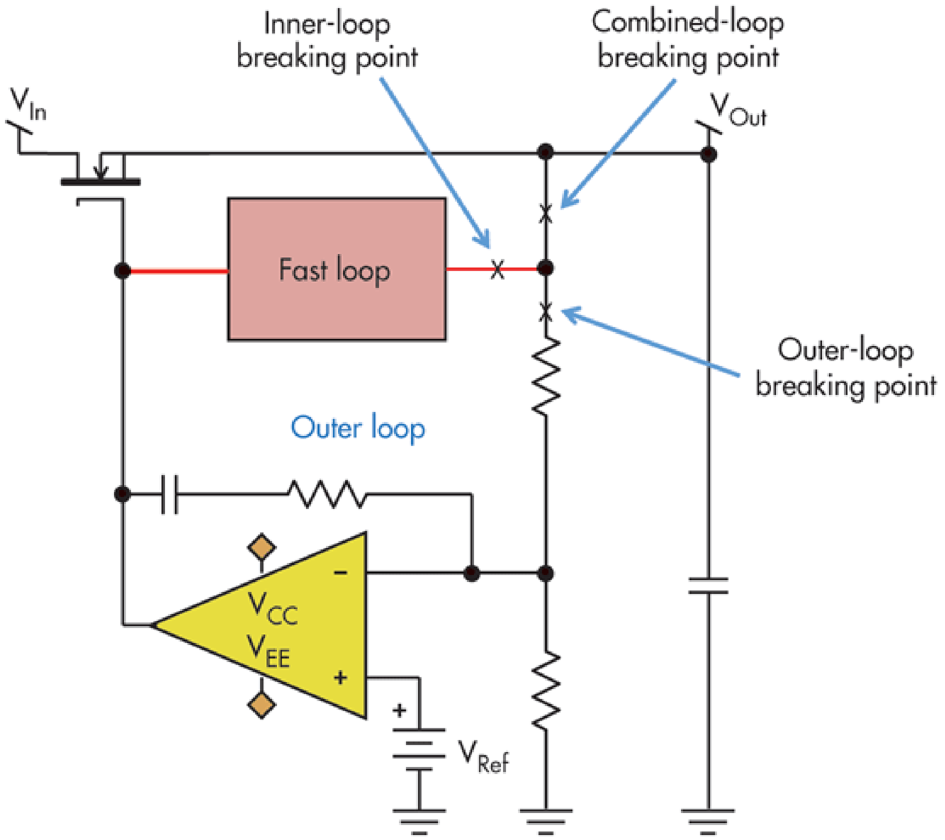
Abstract

Non-Invasive Stability Measurement (NISM) determines closed-loop stability directly from **output-impedance measurements**, eliminating the need for signal injection or internal loop access. Developed over a decade ago and included in the Bode Analyzer Suite from its inception, NISM is now gaining broader adoption as simulators and instruments implement it—driven in part by the rise of topologies that are neither linear nor time-invariant and, therefore, incompatible with traditional Bode-plot methods. Because it works from any impedance measurement and has no inherent frequency limitations, NISM applies across a wide range of circuits, including RF/microwave amplifiers, op-amps, LDOs, multi-phase VRMs, and even input-filter stability.

This work presents—for the first time—the complete mathematical derivation that links measured output impedance to quantitative stability margins. We provide practical guidance for acquiring the required data using high-fidelity measurement tools, and we show how paired ON/OFF measurements separate active-loop behavior from passive Q to reveal the true stability-limiting peaks. Through real-world examples, we demonstrate NISM as a universal, measurement-driven approach for fast, accurate, and fully non-invasive stability evaluation.

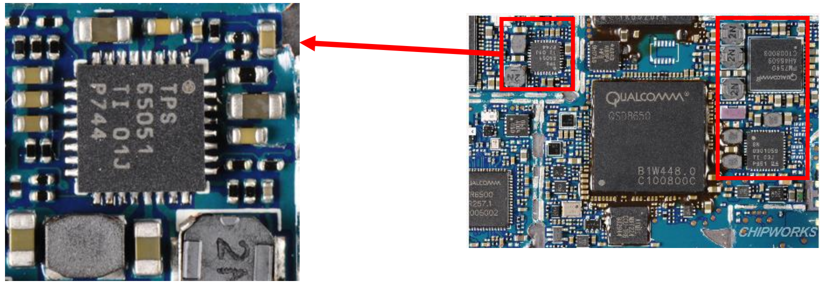
Why Stability Measurement Is Getting Harder

- Small-signal assumptions
- Linear behavior
- Time-invariant systems
- Bode plots don't provide relative stability
- Bode plots don't directly relate to step load which is the focus of power integrity

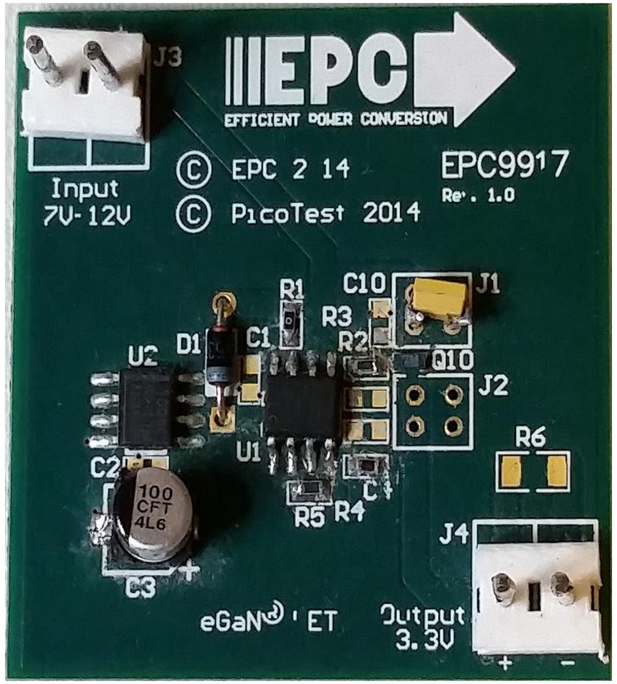


PICOTEST

Problem Statement



- Many devices lack physical or schematic access for loop injection
- Many newer devices have multiple internal (inaccessible) loops
- High-frequency devices (op-amps, ADC buffers, GaN LDOs) exceed practical injection bandwidth

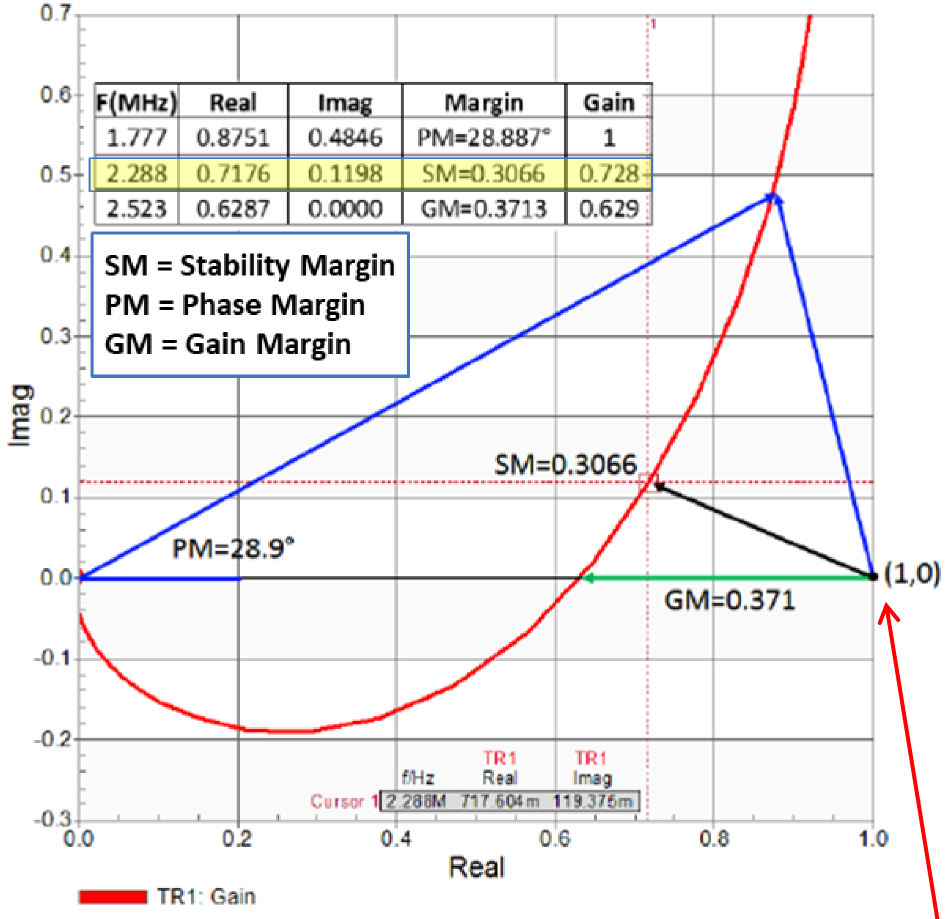


PICOTEST

Closed-Loop Stability

$$Closed_{Loop} = \frac{Open\_Loop}{1 + T}$$

- Stability relates open loop and closed loop data using the loop gain vector T
- The denominator determines how close the loop gain vector T approaches the critical point
- Phase margin assesses the distances between 1,0 and T on the unit circle
- Gain margin assess the distance between 1,0 and T on the horizontal axis
- Stability margin assesses the closest distance between T and 1,0
- Phase margin and Gain margin assure stability, but this is NOT relative. SM represents the closest distance from the singular unstable point. **This is a major reason that we measure the Bode plot AND perform the step-load test**



ONLY this one point is truly *UNSTABLE*

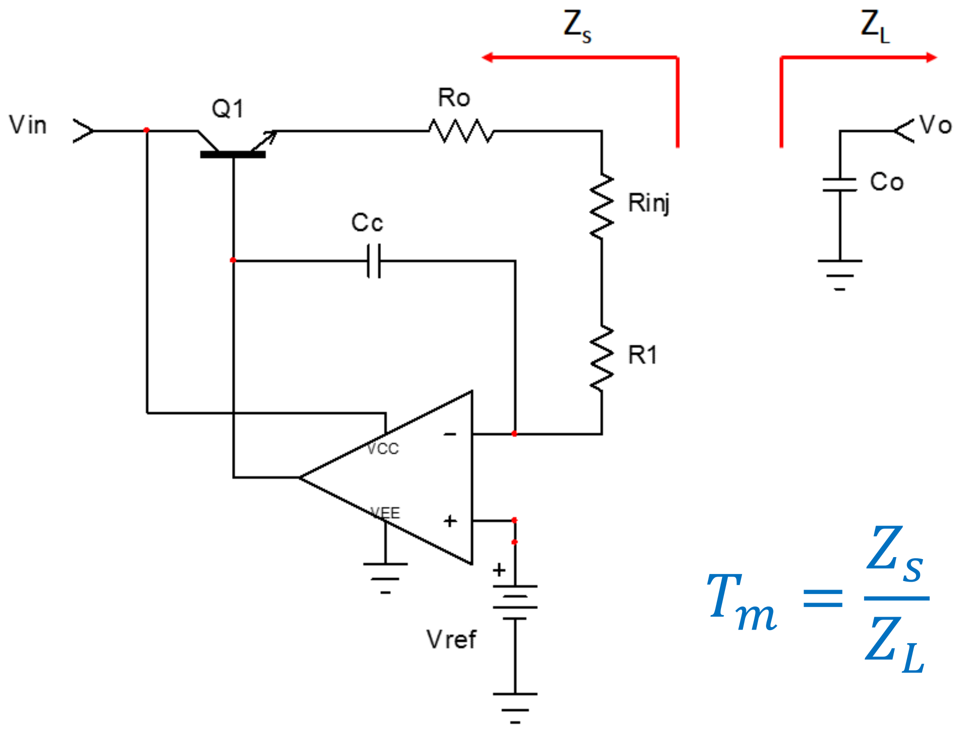
Relative stability is the distance from this point

PICOTEST

Middlebrook's Minor Loop Gain

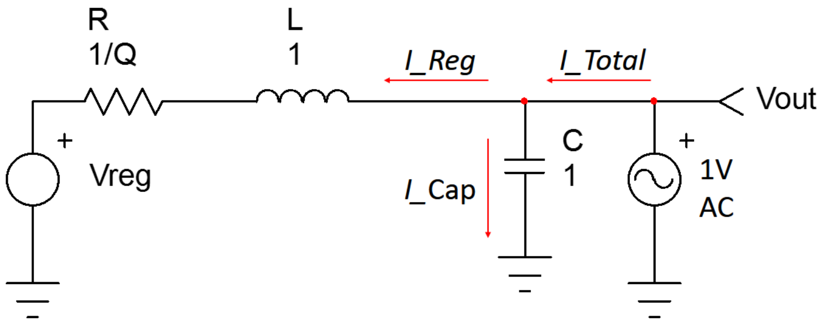
- Fortunately, in 1976 R.D. Middlebrook introduced the concept of Minor Loop Gain
- This minor loop gain theory forms the bridge between impedance and stability
- While this method is helpful, it requires separation at the boundary and two measurements
- Goal: measure stability without lifting wires, cutting traces, or disturbing the circuit

A convenient boundary exists at the output capacitors



$$T_m = \frac{Z_s}{Z_L}$$

Resonant Behavior



$$I\_Reg = \frac{1}{\frac{1}{Q} + k \cdot i}$$

$$mag\_I\_Reg = \frac{1}{\sqrt{\left(\frac{1}{Q}\right)^2 + k^2}}$$

$$I\_Reg\_Re = \frac{1}{\sqrt{\left(\frac{1}{Q}\right)^2 + k^2}} \cdot cos\left(atan(k \cdot Q)\right)$$

$$I\_Reg\_Im = \frac{1}{\sqrt{\left(\frac{1}{Q}\right)^2 + k^2}} \cdot sin\left(atan(k \cdot Q)\right) \cdot i$$

$$I_{total} = \left(\frac{Qk}{\sqrt{Q^2k^2+1} \cdot \sqrt{\frac{1}{Q^2} + k^2}} - k\right) \cdot i + \frac{1}{\sqrt{Q^2k^2+1} \cdot \sqrt{\frac{1}{Q^2} + k^2}}$$

$$Zout\_mag = \frac{(Qk)^2 + 1}{\sqrt{Q^2 + Q^4k^2 + k^2[(Qk)^2+1]^2 - 2(Qk)^2[(Qk)^2+1]}}$$

$$Zout\_phase = atan\left[Qk - k\sqrt{Q^2k^2+1}\sqrt{\frac{1}{Q^2} + k^2}\right]$$

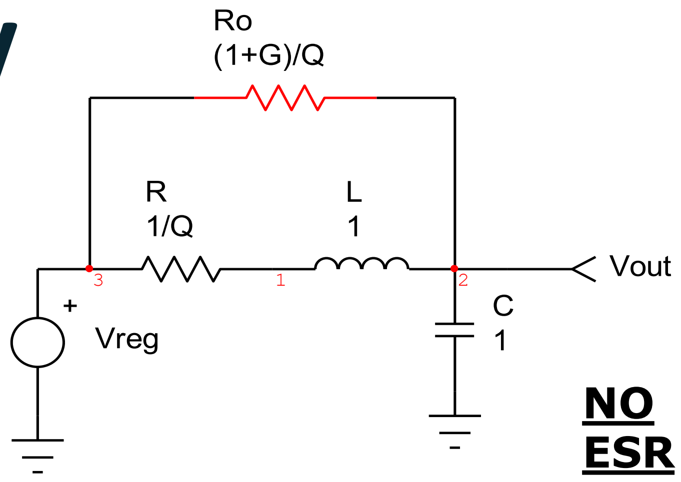
Setting phase to zero and solving for k (resonance)

$$k_z = \frac{\sqrt{Q^2-1}}{Q} = \sqrt{\frac{Q^2-1}{Q^2}} = \sqrt{1 - \frac{1}{Q^2}}$$

This result aligns with Cartwright’s published solution

PICOTEST

Solving for Crossover Frequency



NO ESR

$$Zreg = k \cdot i + \frac{1}{Q} = Z_s$$

$$Zcap = \frac{1}{k \cdot i} = Z_L$$

$$T_m = \frac{Zreg}{Zcap} = \frac{k \cdot i + \frac{1}{Q}}{\frac{1}{k \cdot i}}$$

$$T_m = \frac{|Zreg|}{|Zcap|}$$

T_m=1 (Unity gain)

$$\left|k \cdot i + \frac{1}{Q}\right| = \left|\frac{1}{k \cdot i}\right|$$

$$\sqrt{k^2 + \frac{1}{Q^2}} = \frac{1}{k}$$

$$k = 2\pi f \; for\ normalized\ L\ and\ C = 1$$

$$k_c = \sqrt{\frac{\sqrt{4Q^4 + 1} - 1}{2Q^2}}$$

PICOTEST

Stability Margin

Separating $T_m = \frac{Zreg}{Zcap}$ into real and imaginary terms

$$T_m = \frac{\sqrt{4Q^4+1} + \sqrt{2}Q \cdot i \cdot \sqrt{\frac{\sqrt{4Q^4+1}-1}{Q^2} - 1}}{2Q^2}$$

And applying simple trigonometry

$$PM = \mathrm{atan}\left(\frac{Im}{Re}\right)$$

$$PM = atan\left(\sqrt{\frac{2}{\sqrt{4Q^4+1}-1}}\right)$$

This isn't new and confirms Erickson and Maksimovic published result, so Minor Loop works

Phase Margin Extraction

Separating $T_m = \frac{Zreg}{Zcap}$ into real and imaginary terms

$$T_m = \frac{\sqrt{4Q^4+1} + \sqrt{2}Q \cdot i \cdot \sqrt{\frac{\sqrt{4Q^4+1}-1}{Q^2} - 1}}{2Q^2}$$

And applying simple trigonometry

$$PM = \operatorname{atan}\left(\frac{Im}{Re}\right) \Rightarrow PM = atan\left(\sqrt{\frac{2}{\sqrt{4Q^4+1}-1}}\right)$$

This is not new and confirms Erickson and Maksimovic published result, so Minor Loop works

Note this was all solved using a second order RLC model and without ESR

Getting Rid of the Arctan

$$\frac{d}{dk}atan\big(u(k)\big) = \frac{1}{1+u^2}\cdot\frac{du}{dk}$$

$$u(k) = Qk - k\sqrt{Q^2k^2+1}\sqrt{\frac{1}{Q^2}+k^2}$$

After applying the chain rule, substituting and simplifying, we have a solution for Group Delay based Q, as a function of Q, but we still have the variable k that is in the way

$$T_gQ = \frac{Q^2k\sqrt{\frac{1}{Q^2}+k^2}\left(4Q^2k^2+3Q^4k^4-Q^3\sqrt{Q^2k^2+1}\sqrt{\frac{1}{Q^2}+k^2}+1\right)}{2(Q^2k^2+1)^{\frac{3}{2}}\left(Q^2+k^2+2Q^2\,k^4+Q^4k^2+Q^4k^6-2Q^3k^2\sqrt{Q^2k^2+1}\sqrt{\frac{1}{Q^2}+k^2}\right)}$$

Group Delay and Q

Everything is dependent on Q and it isn't a direct measured parameter

$Q = \pi \cdot T_g \cdot f$ Borrowed from the RF playbook

I had previously determined phase to be → $Zout\_phase = atan\left[Qk - k\sqrt{Q^2k^2+1}\sqrt{\frac{1}{Q^2}+k^2}\right]$

And the closed loop Q from Group Delay is

$$T_gQ = \frac{d}{dk}atan\left[Qk - k\sqrt{Q^2k^2+1}\sqrt{\frac{1}{Q^2}+k^2}\right]\cdot\frac{-k}{2}$$

Tg is available from an output impedance measurement using VNA or Scope

PICOTEST

Eliminating k

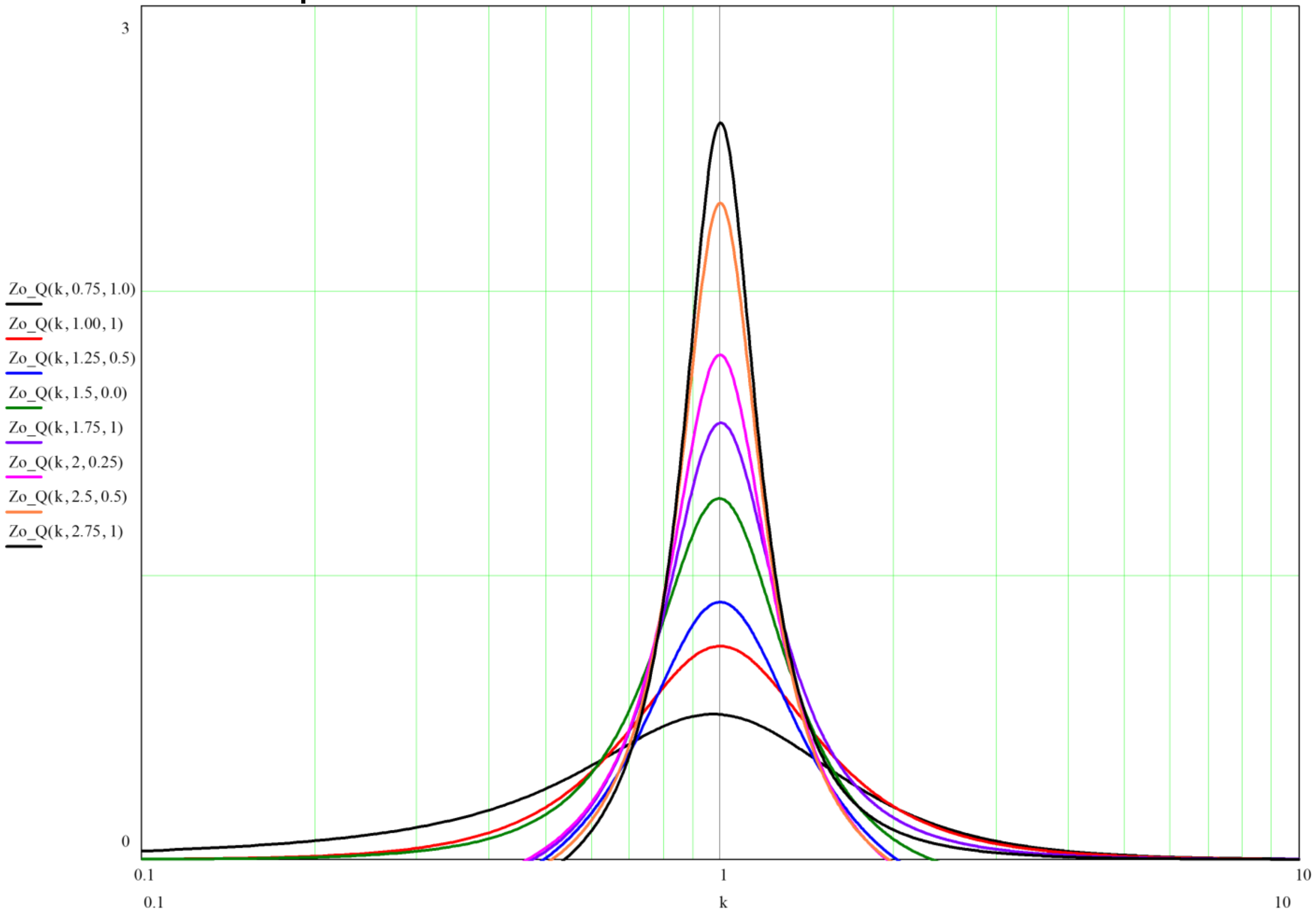
The resonant frequency of TgQ =1 independent of Q

This result is significant — and becomes even more important when ESR is included

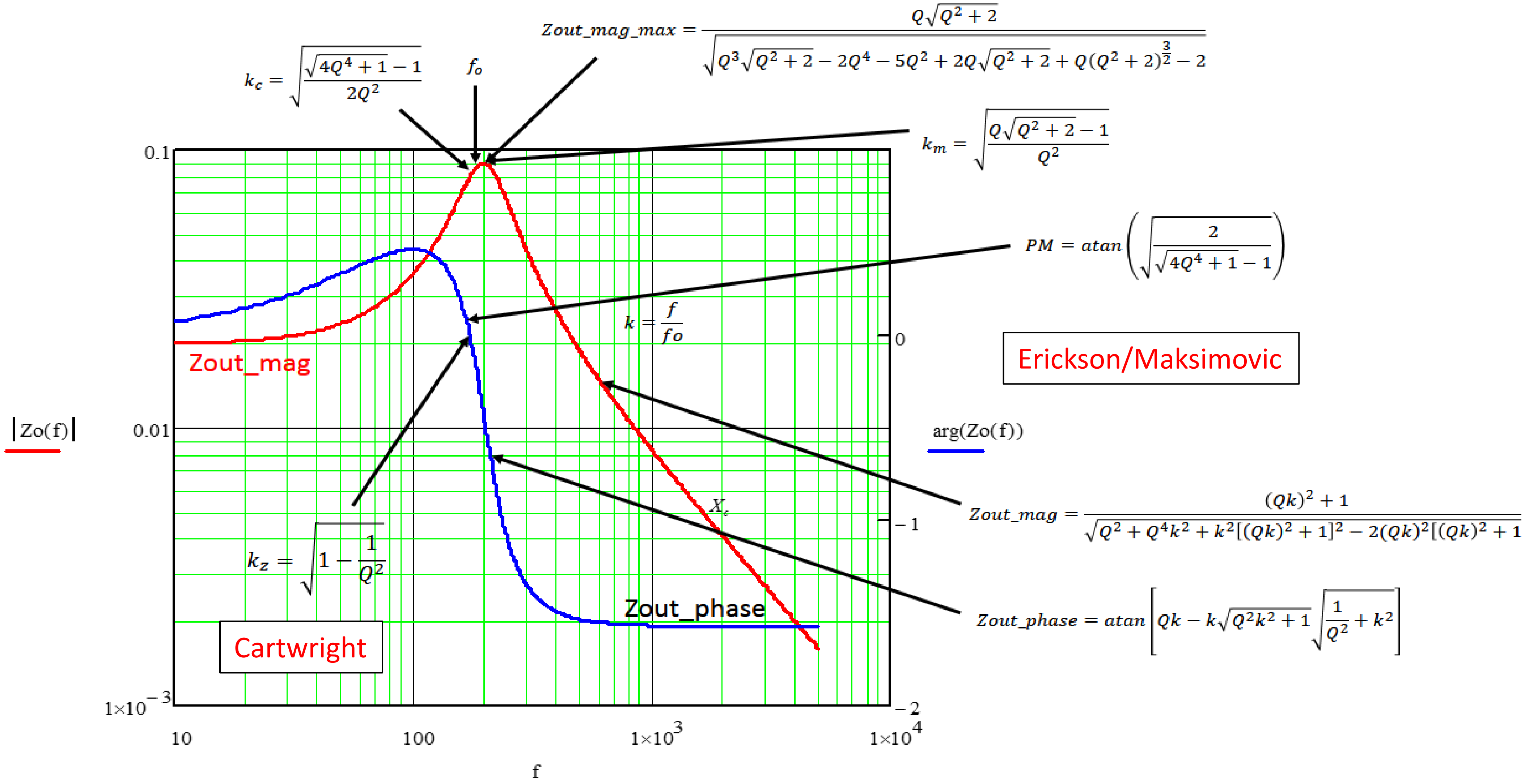
Setting the first derivative of TgQ to zero and solving for k results in…… 1.0

Allowing TgQ to be reduced to

$$TgQ = \frac{2Q + \frac{1}{Q}}{2\left(\frac{1}{Q^2} + 1\right)}$$

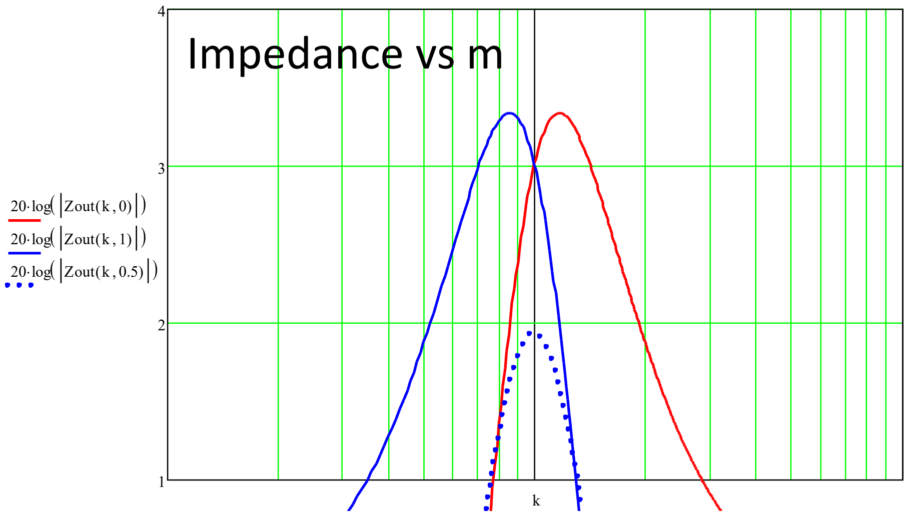


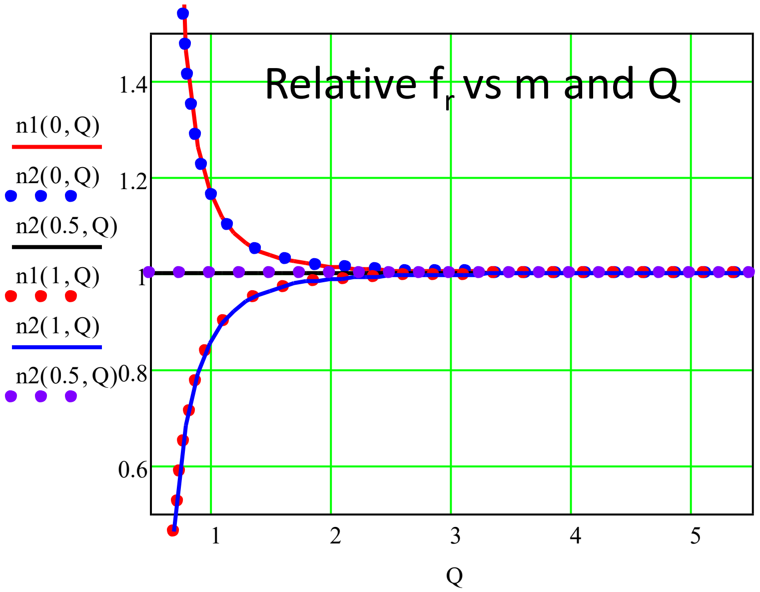
Mathematical Contribution

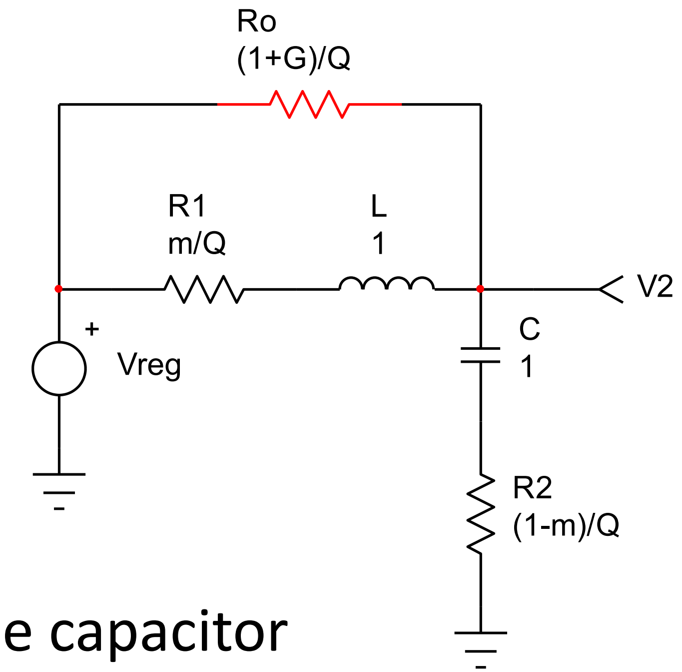


But We Can't Ignore ESR!!

Introduce parameter m to partition total Q between regulator and capacitor ESR







As the distribution of the Q moves between the regulator and the capacitor ESR, the impedance resonance shifts in frequency relative to the Q resonance. The impedance magnitude also changes.

Solving as Before With m Included

$$TgQ(Q,m) := \frac{2 \cdot Q^5 + 4 \cdot Q^3 \cdot m^2 - 4 \cdot Q^3 \cdot m + Q^3 + 2 \cdot Q \cdot m^4 - 4 \cdot Q \cdot m^3 + 3 \cdot Q \cdot m^2 - Q \cdot m}{2 \cdot Q^4 + 4 \cdot Q^2 \cdot m^2 - 4 \cdot Q^2 \cdot m + 2 \cdot Q^2 + 2 \cdot m^4 - 4 \cdot m^3 + 2 \cdot m^2}$$

And finding the frequency of the maximum impedance

$$k_m := \frac{\sqrt{\left[Q^2 - 2 \cdot Q^2 \cdot m + \sqrt{\left(Q^2 + m^2 - m\right)^2 \cdot \left(Q^4 + 2 \cdot Q^2 \cdot m^2 - 2 \cdot Q^2 \cdot m + 2 \cdot Q^2 + m^4 - 2 \cdot m^3 - m^2 + 2 \cdot m\right)}\right] \cdot \left(Q^4 + 2 \cdot Q^2 \cdot m^2 - 4 \cdot Q^2 \cdot m + 2 \cdot Q^2 + m^4 - 2 \cdot m^3 + 2 \cdot m - 1\right)}}{Q^4 + 2 \cdot Q^2 \cdot m^2 - 4 \cdot Q^2 \cdot m + 2 \cdot Q^2 + m^4 - 2 \cdot m^3 + 2 \cdot m - 1}$$

Solving for Phase Margin

$$Z_s = \left(k \cdot i + \frac{m}{Q}\right) \qquad Z_L = \left(\frac{1}{k \cdot i} + \frac{1-m}{Q}\right)$$

$$T_m = \frac{Z_s}{Z_L} = \frac{k \cdot i + \frac{m}{Q}}{\frac{1}{k \cdot i} + \frac{1-m}{Q}}$$

$$k_c = \sqrt{\frac{\sqrt{4(Q^4 + m^2 - m) + 1} - 2m + 1}{2Q^2}}$$

$$PM = atan\left[\frac{\sqrt{2}\left(2Q^2 m - 3m + 2m^2 - m\sqrt{4(Q^4 + m^2 - m) + 1} + \sqrt{4(Q^4 + m^2 - m) + 1} + 1\right)}{2\sqrt{\sqrt{4(Q^4 + m^2 - m) + 1} - 2m + 1} \cdot (Q^2 + m^2 - m)}\right]$$

Matlab Solver – 2 equations and 2 unknowns

Seed Values	Measurements	Constants
$m := 0.3$	$Q\_Tg := 0.642$	$f_o := 199.5$
$Q := .8$	$k_m := \frac{218.5}{f_o}$	

Given

$$k_m = \frac{\sqrt{\left[Q^2 - 2 \cdot Q^2 \cdot m + \sqrt{(Q^2 + m^2 - m)^2 \cdot (Q^4 + 2 \cdot Q^2 \cdot m^2 - 2 \cdot Q^2 \cdot m + 2 \cdot Q^2 + m^4 - 2 \cdot m^3 - m^2 + 2 \cdot m)}\right]}}{\sqrt{Q^4 + 2 \cdot Q^2 \cdot m^2 - 4 \cdot Q^2 \cdot m + 2 \cdot Q^2 + m^4 - 2 \cdot m^3 + 2 \cdot m - 1}}$$

$$Q\_Tg = \frac{1}{2} \frac{Q \cdot (2Q^2 + 2m^2 - 2m + 1) \cdot (Q^2 + m^2 - m)}{(Q^2 + m^2) \cdot (Q^2 + m^2 - 2m + 1)}$$

$$\text{Find}(m, Q) = \begin{pmatrix} 0.25 \\ 1 \end{pmatrix}$$

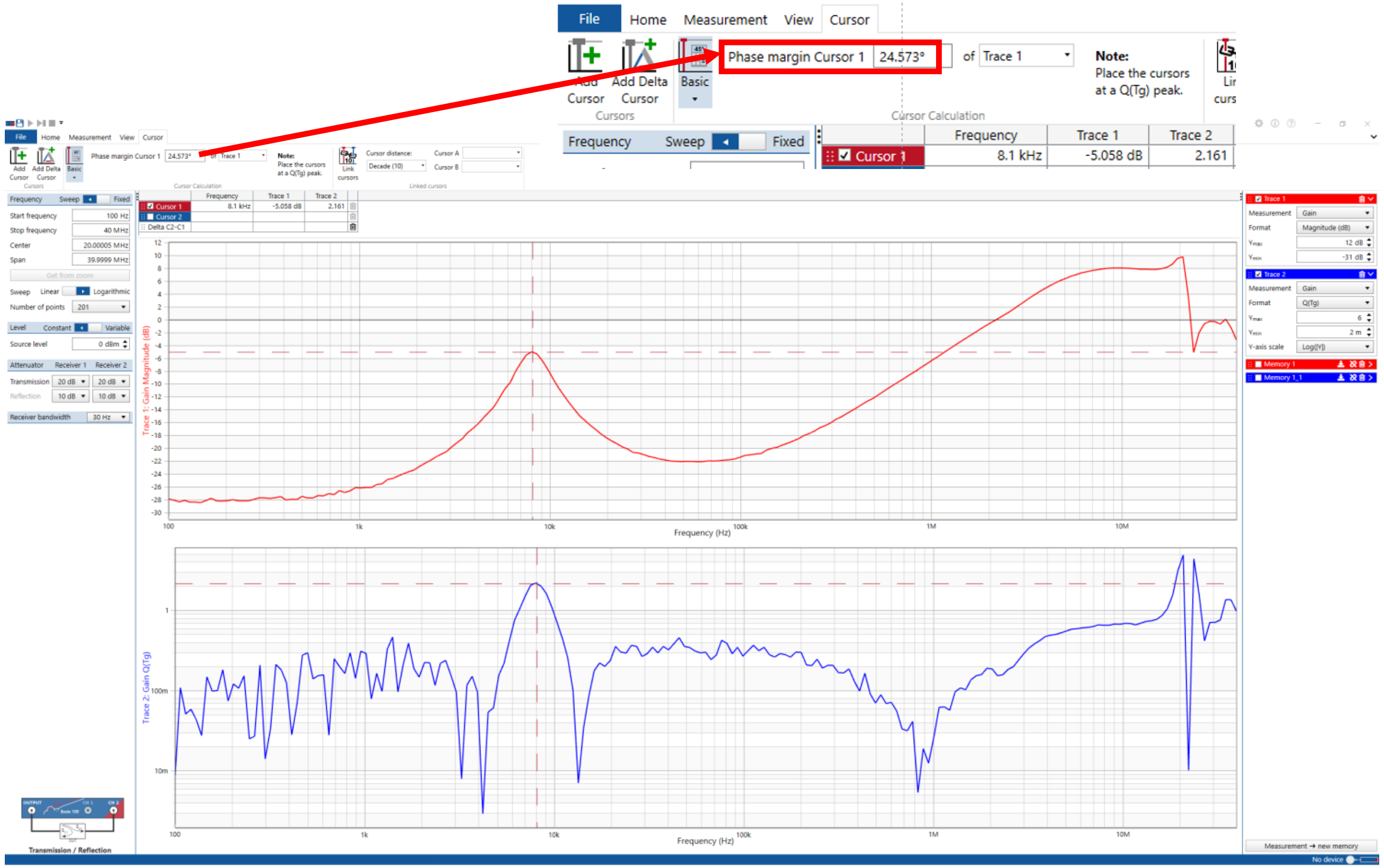
Reducing this is quite messy and complex, but this is the exact result in a solvable form.

PICOTEST

NISM Workflow

Different instruments will operate differently. For the Bode 100 and Bode 500, NISM is located in the cursor menu

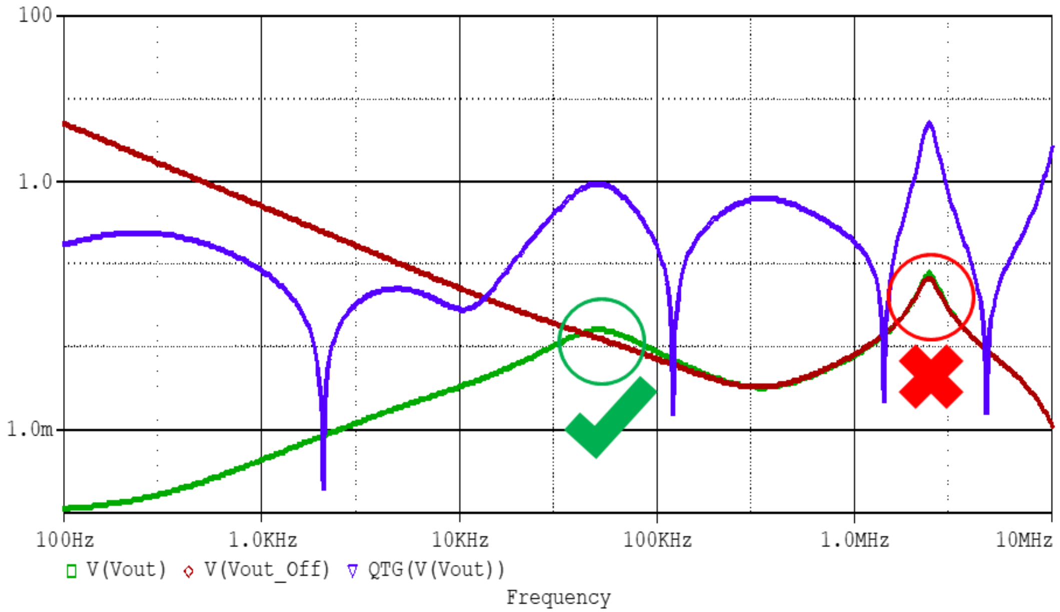
- Measure impedance magnitude and Qtg using any appropriate 1-port, 2-port, or 3-port setup with a supporting simulator, VNA, or Scope
- Place cursors at the impedance magnitude peak and Qtg peak as instructed
- Read the PM as displayed in the software
- NOTE that it is possible that there will be more than one peak

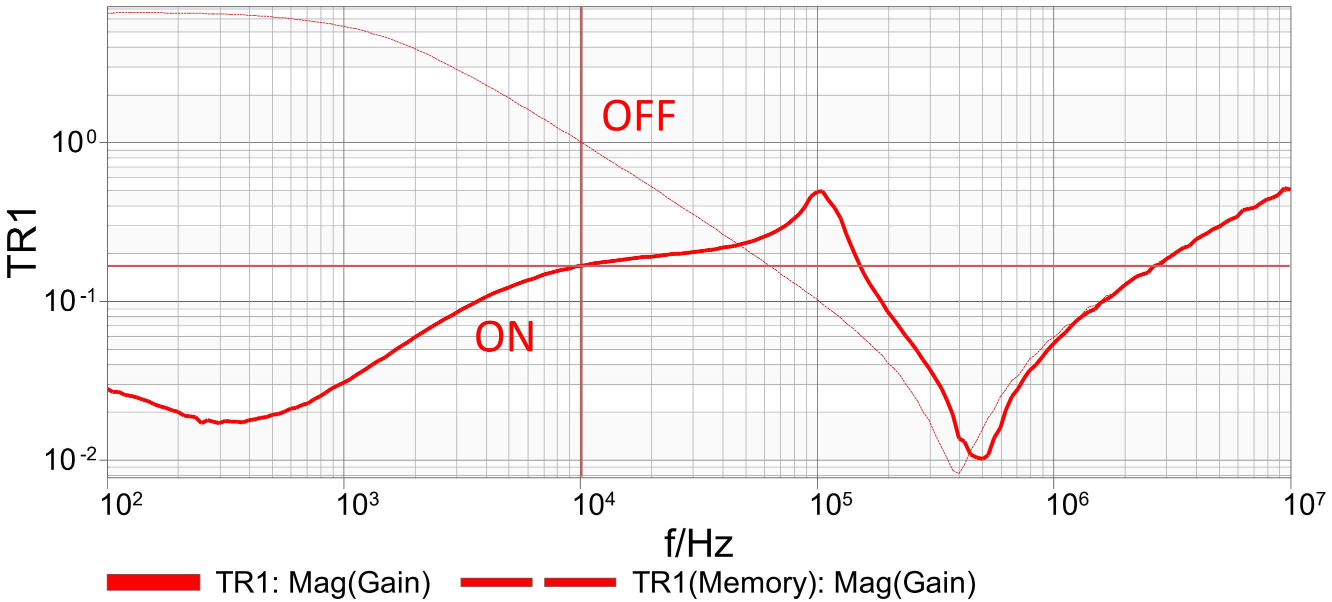


PICOTEST

Identifying Which Peak

- Stability related peaks will ONLY appear with the power ON and will not be present with the power OFF
- There can be more than one peak
- OFF-state peaks indicate passive resonances, not stability-limiting behavior

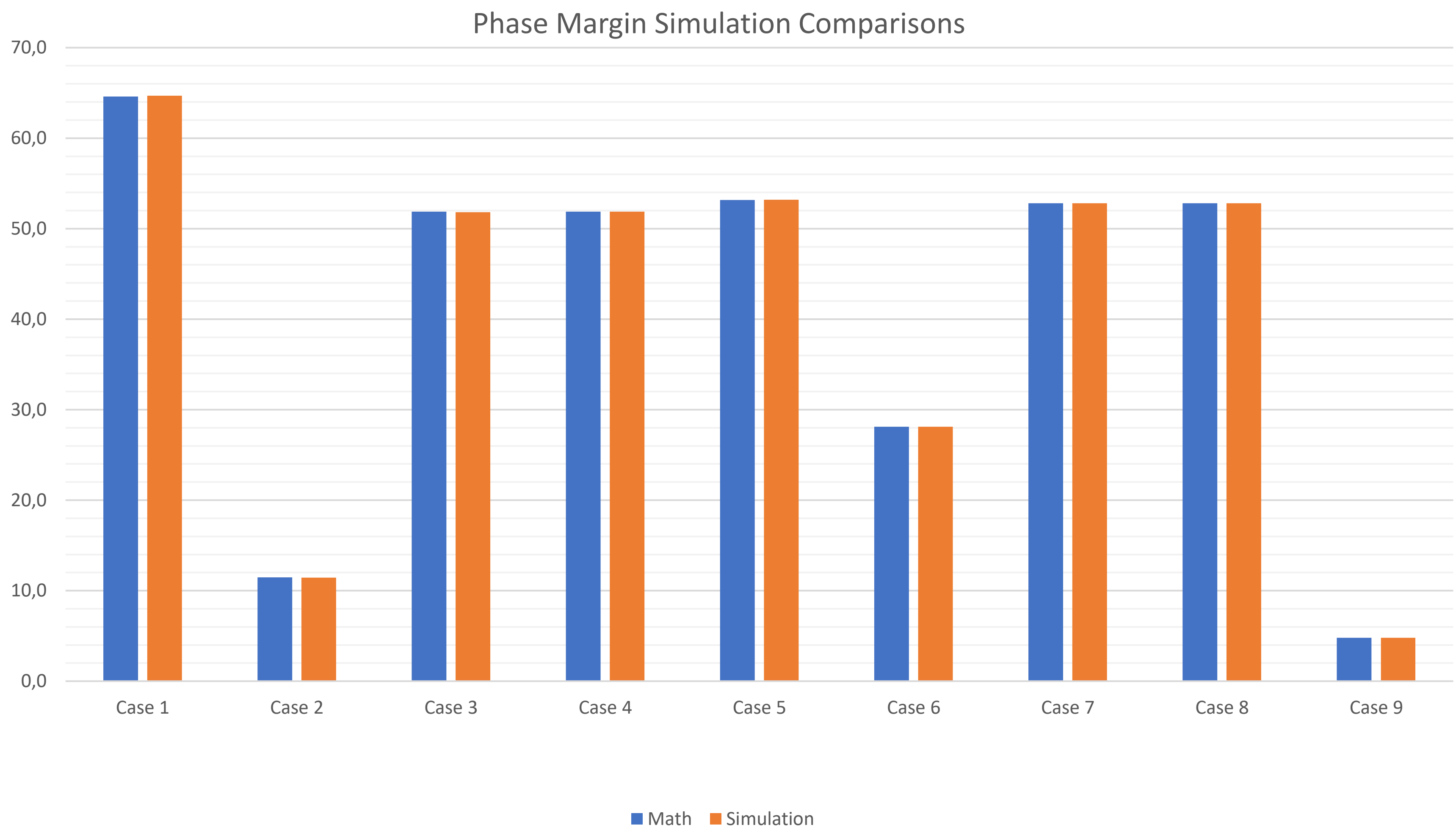




PICOTEST

Simulation Validation Phase Margin

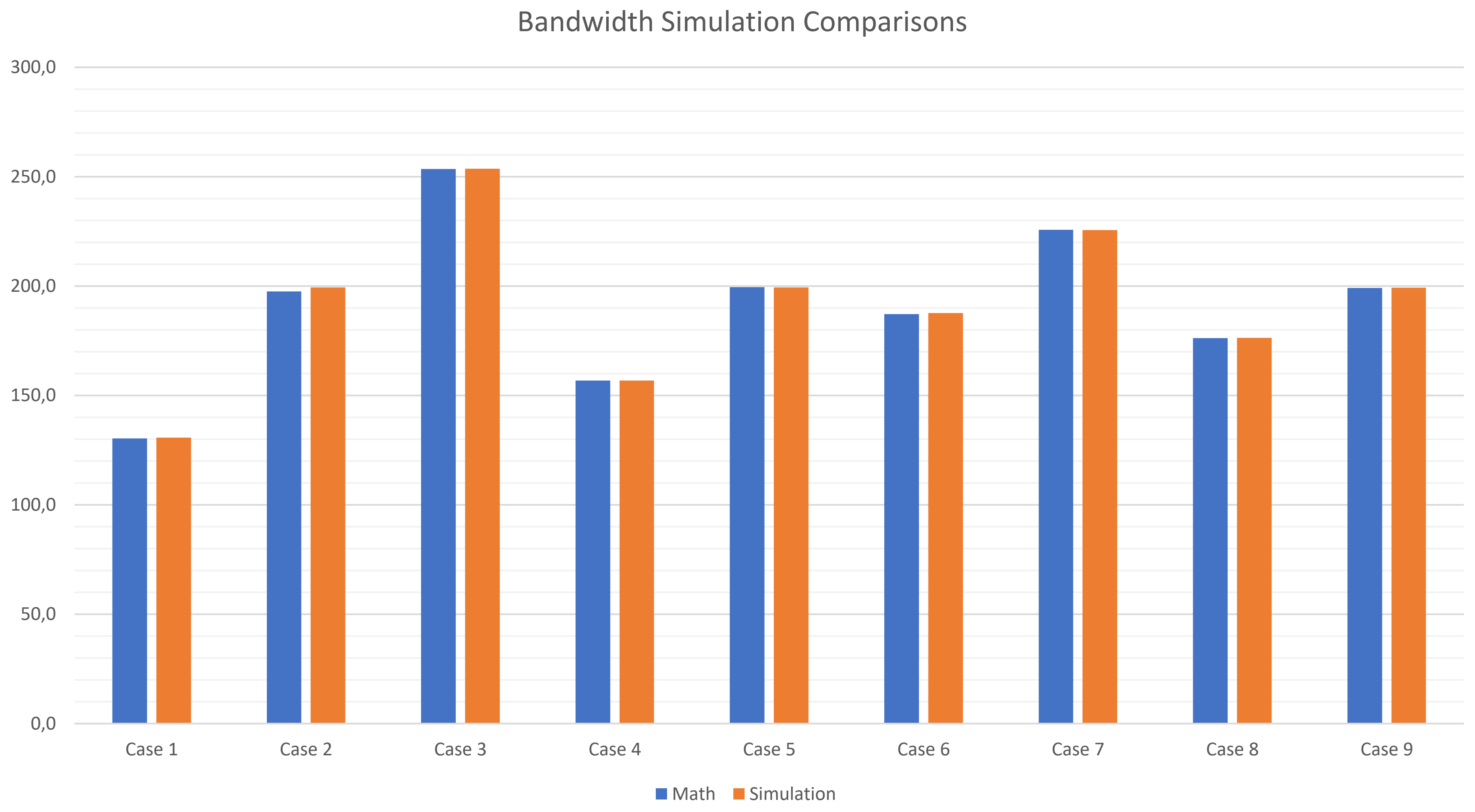
	Math	Simulation	abs error	% error
Case 1	64.6	64.7	-0.090	0.14%
Case 2	11.5	11.4	0.020	0.17%
Case 3	51.9	51.8	0.052	0.10%
Case 4	51.9	51.9	0.000	0.00%
Case 5	53.2	53.2	-0.020	0.04%
Case 6	28.1	28.1	0.000	0.00%
Case 7	52.8	52.8	-0.010	0.02%
Case 8	52.8	52.8	0.000	0.00%
Case 9	4.8	4.8	0.010	0.21%



PICOTEST

Simulation Validation Crossover Frequency

	Math	Simulation	abs error	% error
Case 1	130.4	130.7	-0.260	0.20%
Case 2	197.5	199.4	-1.900	0.95%
Case 3	253.5	253.6	-0.100	0.04%
Case 4	156.8	156.8	0.000	0.00%
Case 5	199.5	199.4	0.100	0.05%
Case 6	187.2	187.6	-0.400	0.21%
Case 7	225.7	225.6	0.100	0.04%
Case 8	176.2	176.3	-0.100	0.06%
Case 9	199.2	199.2	-0.050	0.03%



PICOTEST

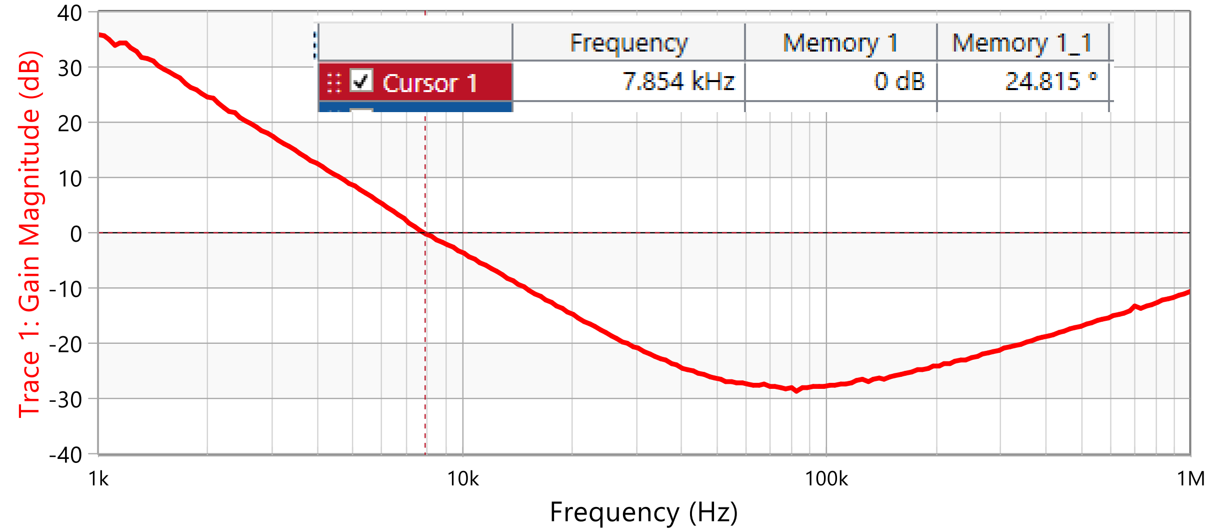
Applicability

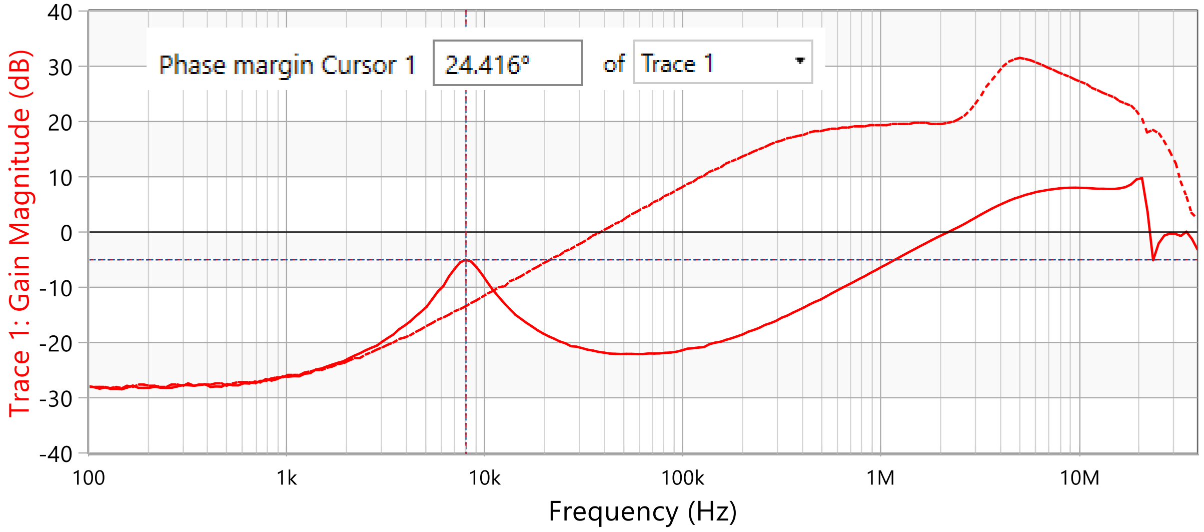
- Linear and switching regulators: LDOs, VRMs, POL converters, multi-phase regulators and PMICs.
- Amplifiers and buffers: Op-amps, ADC drivers, RF/microwave amplifiers, and high-speed line drivers.
- Filters and input stages: Input-filter stability, EMI filters, and grid-connected converters using impedance criteria.
- Wide frequency range: No inherent frequency limit—applies from low-frequency power integrity up into RF and microwave domains.

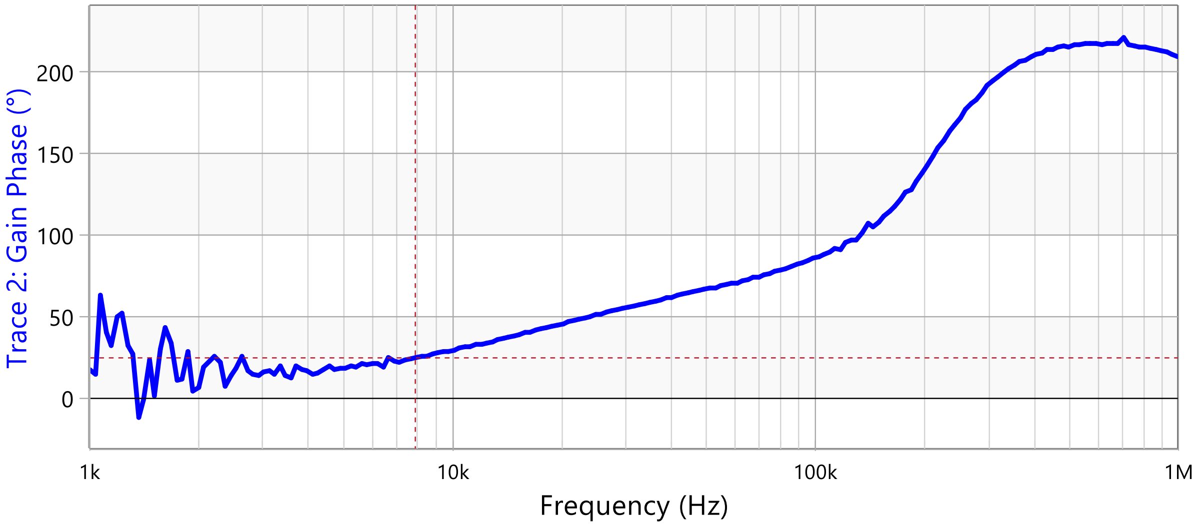
PICOTEST

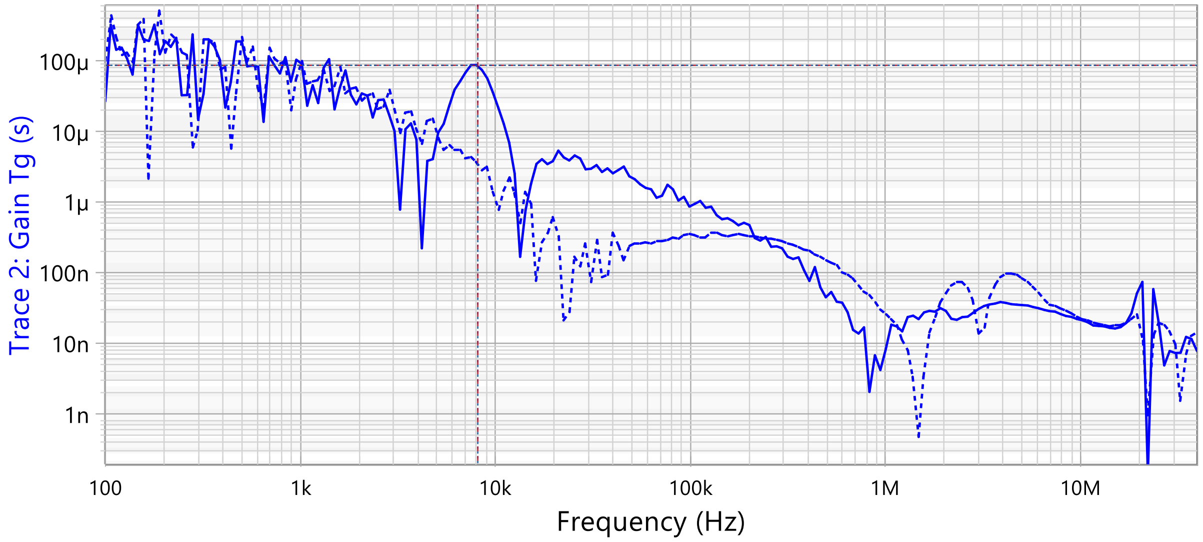
Example: LM317

	NISM	Bode Plot PM
Measured	24.4 deg	24.8 deg



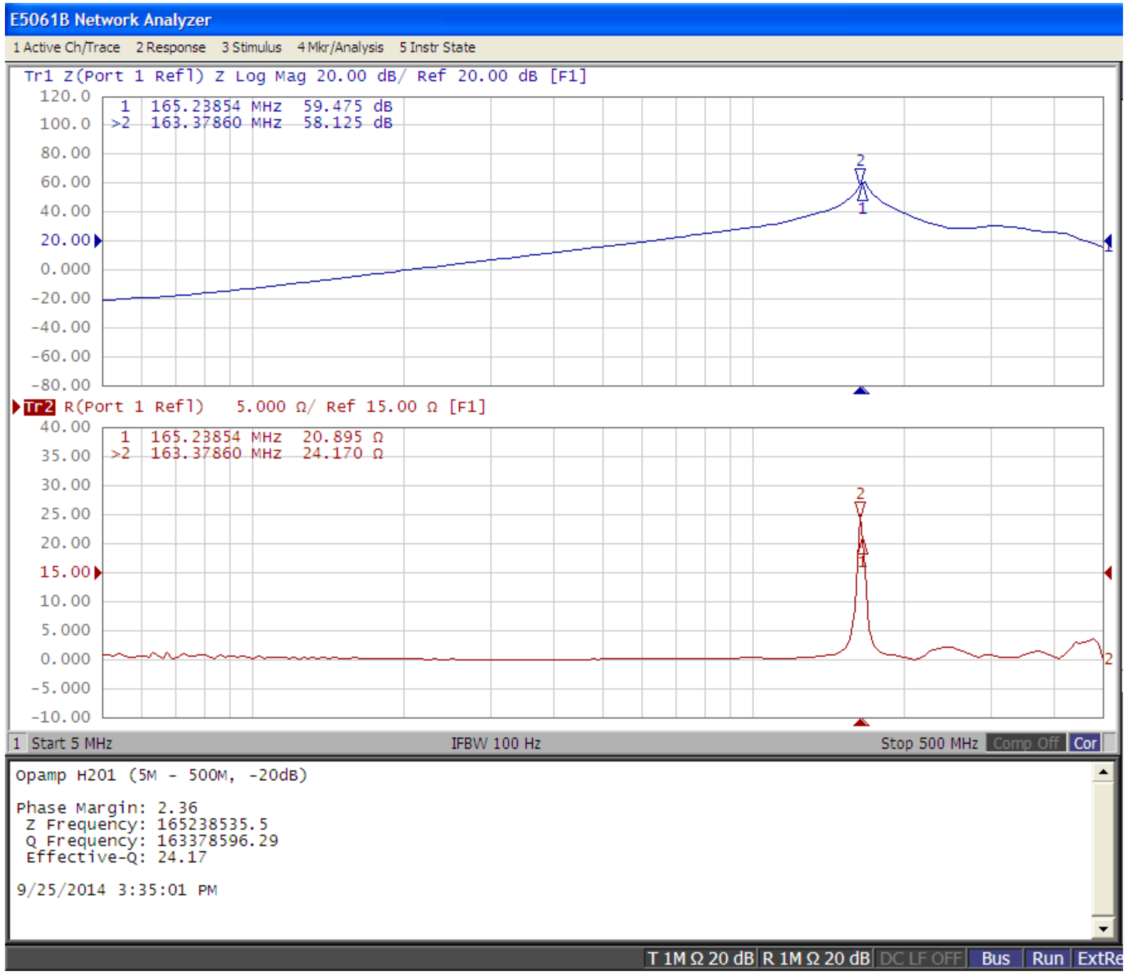




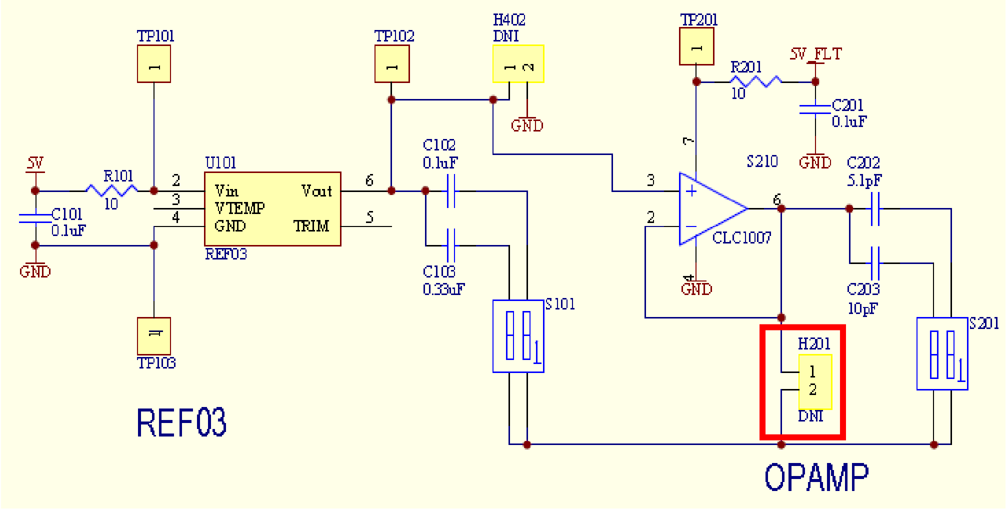


PICOTEST

Example: High-Speed Op-Amp



- Using the Keysight E5061B VNA with the NISM software, we were able to test the stability of this 245 MHz op-amp
- The big WOW is that we obtained the (very poor) phase margin from the impedance measurement using NISM (just about 2 degrees)
- This is a great capability; to be able to accurately assess stability at 100s of MHz or higher without lifting any wires (which would interfere with the measurement)



Example: Line Injector (RF Amplifier)

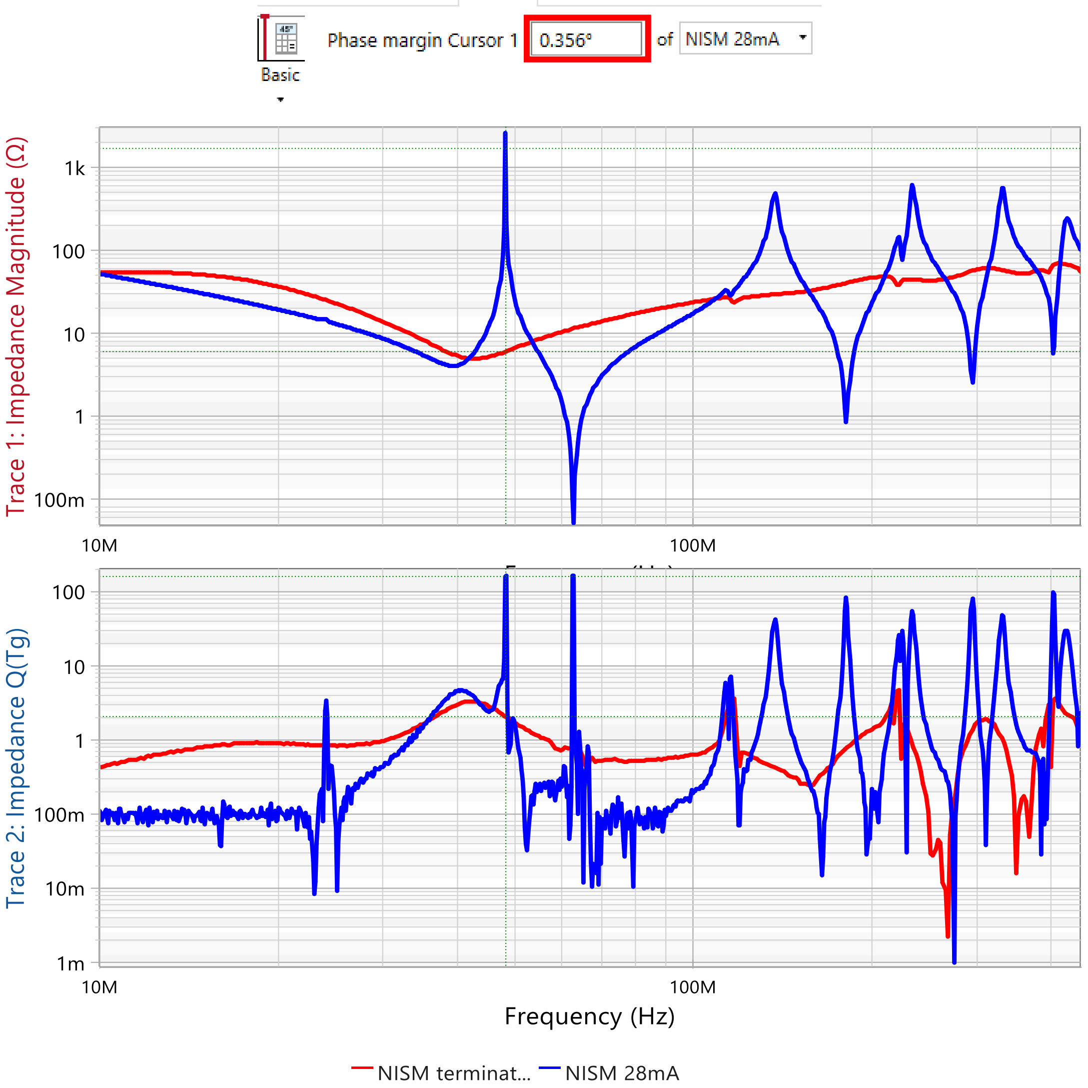
- We had some customer complaints that the J2120A was oscillating.
- The J2120A doesn't have a control loop…??

When the output is disabled in most signal generators, a relay is opened, leaving the modulation cable unterminated.

NISM was applied at the modulator input with the J2120A biased to 28mA, the signal generator enabled and disabled.

The results clearly show near 0 degree margin with the unterminated cable. This is evident in the blue trace, as well as the reflections from the open cable standing waves.

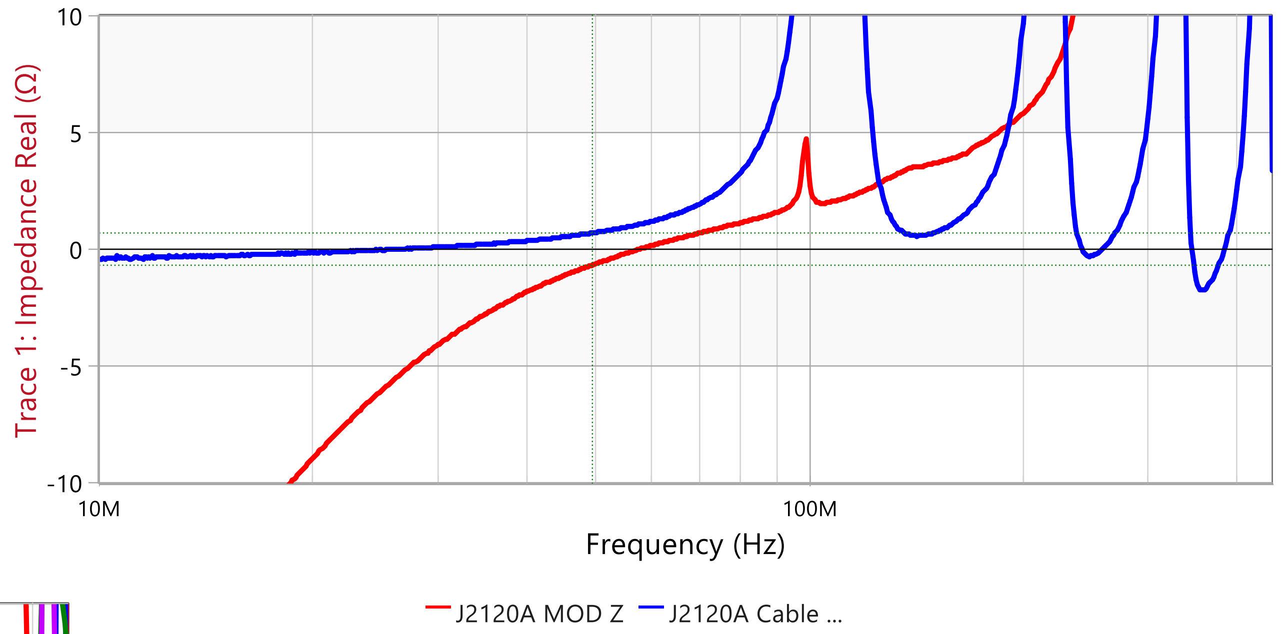
With the signal generator enabled, the far end of the cable is terminated into 50 Ohms by the signal generator, the standing waves are no longer evident, and the peak is gone, indicating good stability.

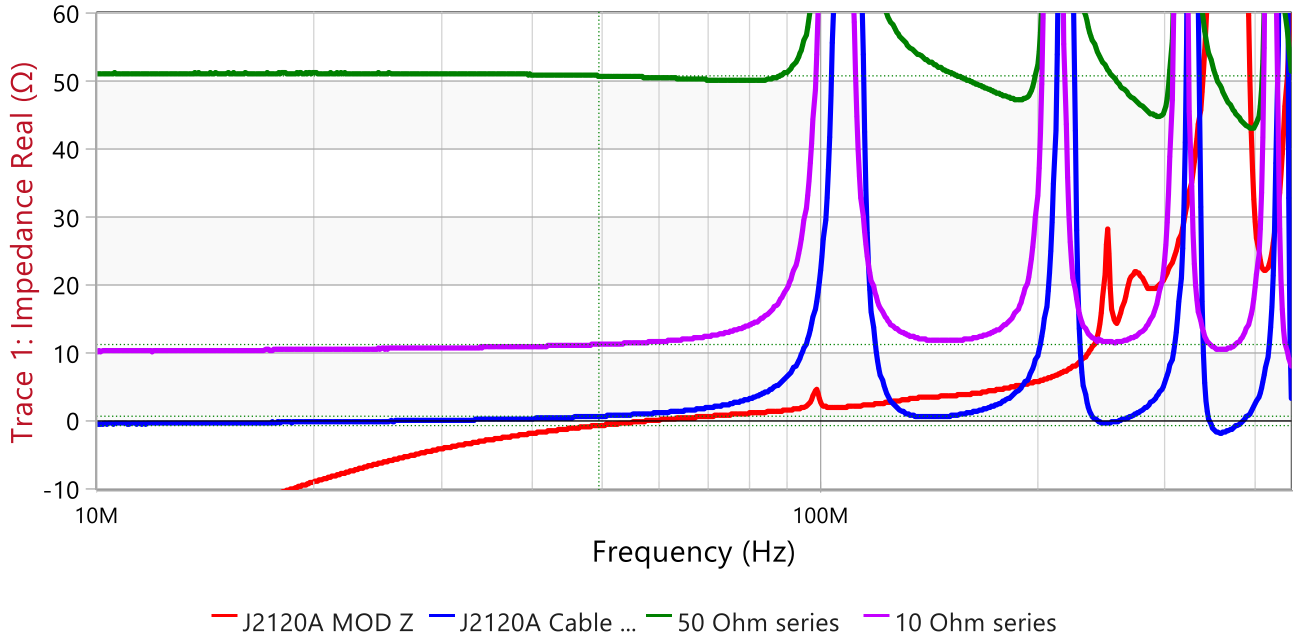


PICOTEST

How'd That Happen?

The J2120A as a biased RF amplifier has a negative resistance region, seen at the modulation port. The sum of the real parts of the negative resistance and the cable resistance is equal to zero at about 49MHz. This defines infinite Q, which defines it as an oscillator.

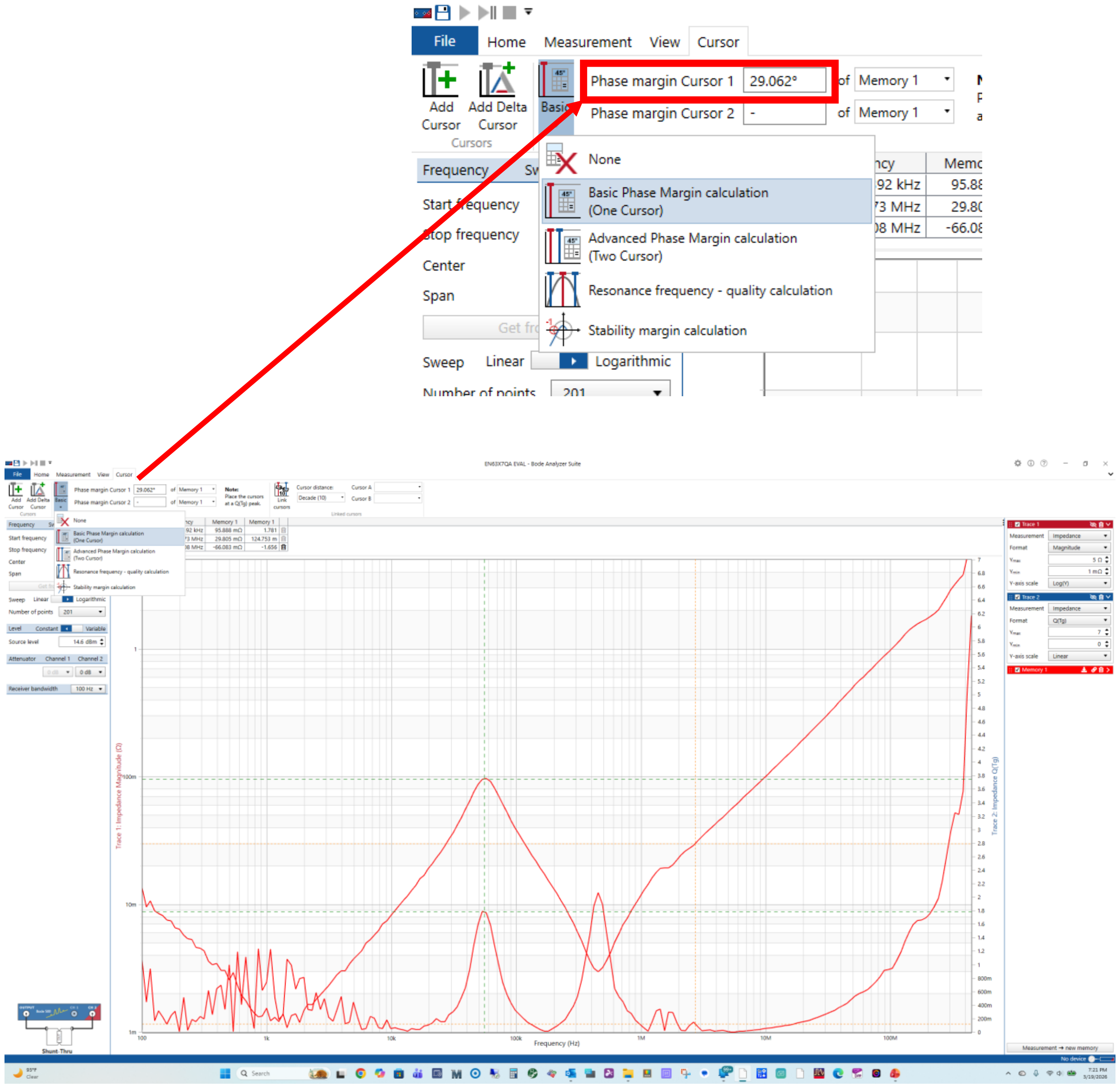




When the cable is terminated into 50 ohms, the impedance separation is much greater, with a positive net real resistance, so it is stable when connected to the unterminated coaxial cable. The resonance of the cable defines the frequency of oscillation, so different length cables will oscillate at different frequencies.

NISM in Industry Today

- NISM has been integrated in the Bode 100 since its inception (~2012)
- NISM is integrated into EDA Tools: Cadence PSpice and Keysight ADS
- There is a standalone version of NISM (.exe)
- Now implemented in commercial VNAs, oscilloscopes, and simulators as a cursor-based function
- Growing ecosystem: Application notes, technical notes, and built-in NISM features are steadily expanding including the time domain equivalent SEPIA (Scope Enabled Power Integrity Analysis) available in oscilloscopes



Video: How to Quantify Flat Impedance and Control Loop Stability with PSpice
https://www.dropbox.com/scl/fi/nf3zyxrgkn7lm1rgixb69/How-to-Quantify-Flat-Impedance-and-Control-Loop-Stability-with-PSpice.mp4?rlkey=bggyaxsorcqqp8yoajcgh1i9c&dl=0

Why Academic Interest Matters

- Active research at Cambridge is extending NISM's theory, validation, and application scope while considering higher order and non-linear systems.
- Peer-reviewed publications are a prerequisite for NISM to become part of industry standards and textbooks.
- When NISM appears in courses and graduate projects, it becomes part of the next generation's default toolkit.
- Accredited mathematical validation helps overcome skepticism from engineers trained exclusively on Bode plots.

Academic Interests and Future Directions

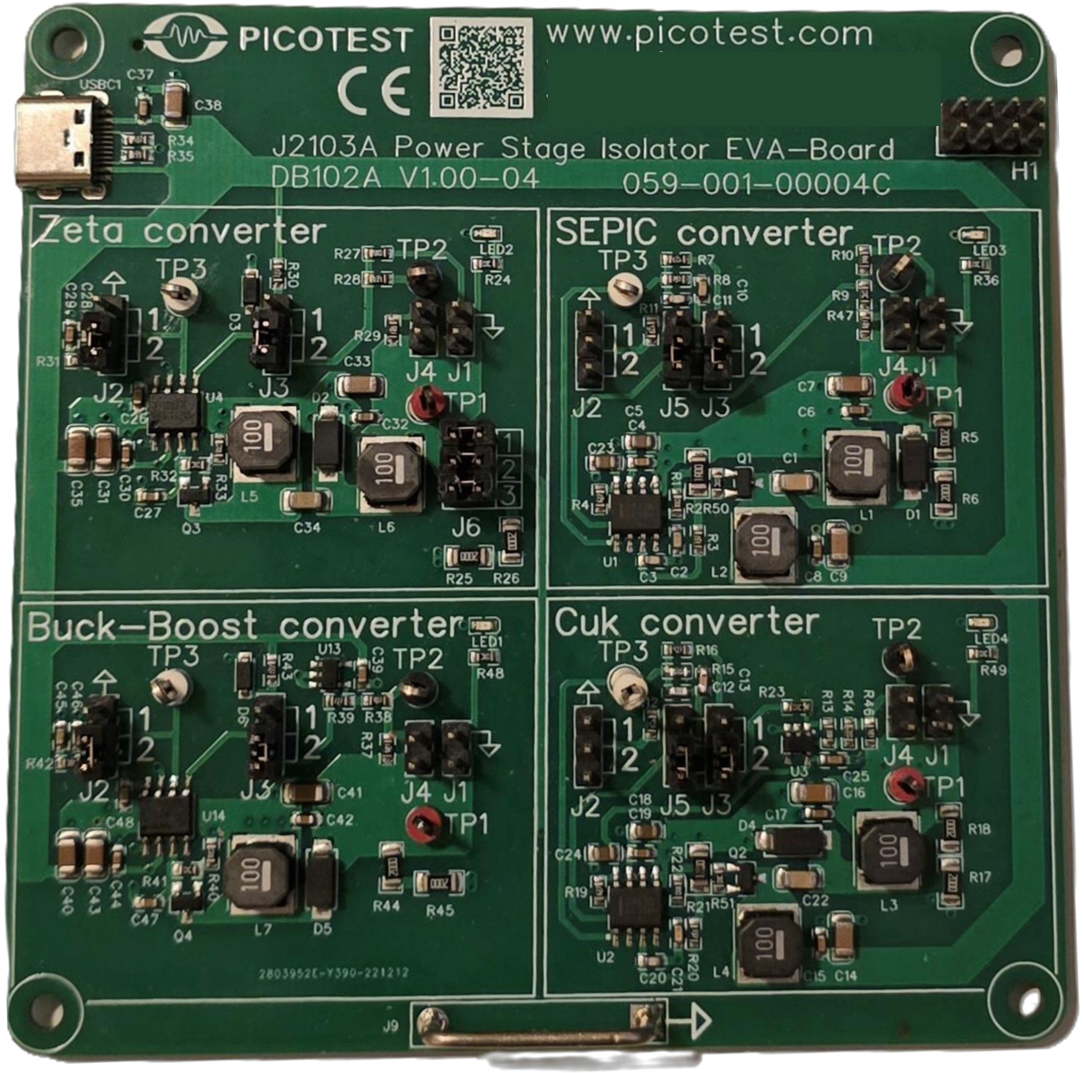
UNIVERSITY OF CAMBRIDGE

- **Current Research**
- **Power System Stability Analysis**
  - Impedance-based stability
  - Small-signal modeling of converters
  - Resonance analysis
- **Methodology**
- **Modeling-driven Approach**
  - RLC-equivalent modeling
  - Frequency-domain analysis (Nyquist / Root locus / Impedance)
  - RF theory
- **Future Direction**
- **Power Converter Design**
- **Power Integrity (PI) Analysis**

What I Hope to Research Related to NISM

UNIVERSITY OF CAMBRIDGE

- **Study of NISM in High-Order Systems**
- **Applicability**
  - Can NISM be applied to all high-order systems?
  - Scope and limitations of the method
- **Accuracy**
  - Does NISM maintain accuracy in high-order and strongly coupled systems?
- **Connection to Stability Metrics**
  - Can NISM be related to other stability indicators?



Evolution of Stability Theory

- Complete mathematical derivation linking output impedance and stability margins, including ESR and Q partitioning.
- NISM extends this lineage by providing a non-invasive path from impedance to stability margins without separation.
- Modern power systems are often nonlinear and time-varying, where traditional Bode plots lose value.
- Time-domain methods like SEPIA complement NISM, offering a parallel evolution in the step-response domain.

PICOTEST

Contributions

- Complete mathematical derivation linking output impedance and stability margins, including ESR and Q partitioning.
- A non-invasive workflow that uses standard impedance measurements—no loop access, no injection, no rewiring.
- NISM embedded in VNAs, Bode analyzers, scopes, and simulators, making it accessible in everyday workflows.

PICOTEST

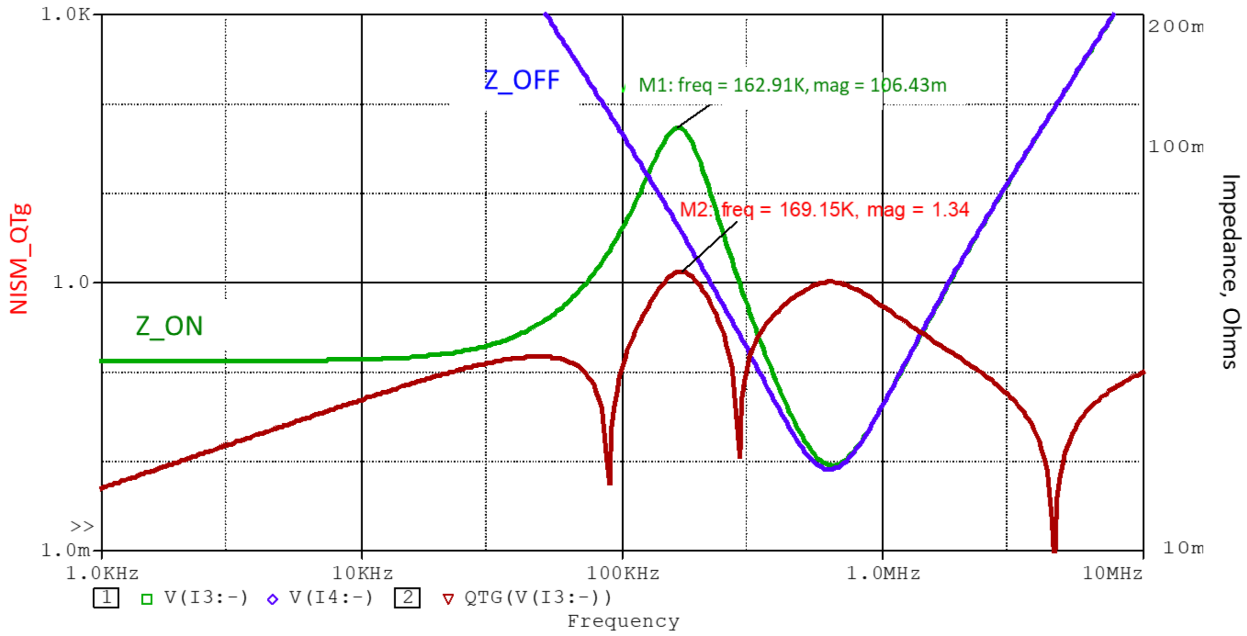
Why Isn't It More Mainstream

- Bode plots are deeply embedded in engineering education, tools, and culture
- Teams have established procedures, fixtures, and sign-off criteria built around loop injection
- Until recently, there was a lack of accredited mathematical validation and academic endorsement
- Adopting NISM requires updating training, documentation, and sign-off criteria, which organizations resist
- Output impedance data is limited and not available in data sheets. Customers don't request it, so part manufacturers don't include it. Most manufacturer's don't even know how to test it.
- Many engineers are simply unaware that Nyquist-accurate stability can be obtained from impedance alone

What I Shared Today

- How impedance maps to stability
- Why ON/OFF matters
- How ESR shifts the peaks
- That NISM is now academically validated and industry-ready

Zout OFF, Zout ON and NISM QTg Data

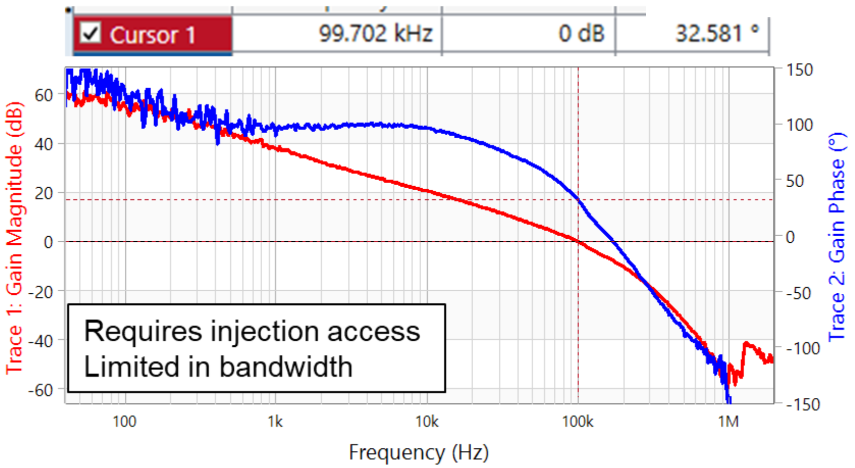


Key Takeaways

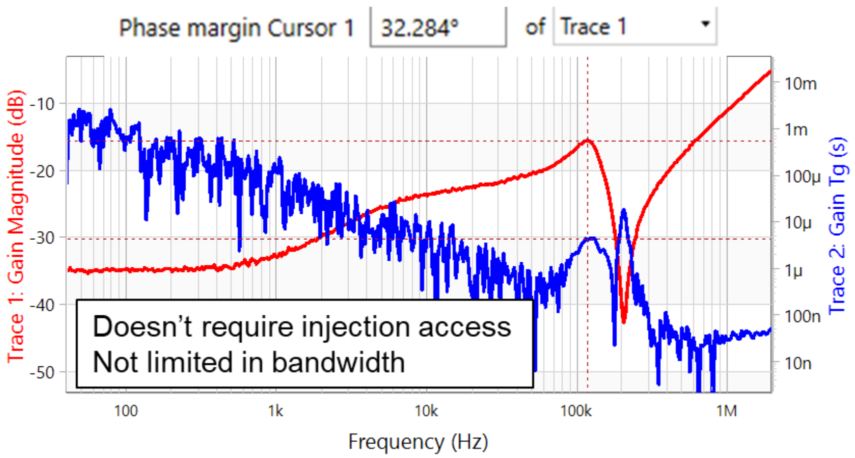
NISM provides Nyquist-level stability without access to the feedback loop

- Universal and non-invasive across regulators, amplifiers, filters, and PDNs
- Backed by a full derivation, ESR-aware modeling, and close agreement with simulation and Bode plots
- As systems become faster, denser, and less accessible, non-invasive stability measurement becomes essential, not optional

Traditional Bode Plot



Impedance Based



0.3 degree difference

PICOTEST

References and Additional Reading

1. Middlebrook, R. D. (1975). Measurement of loop gain in feedback systems. International Journal of Electronics, 38(4), 485–512.
2. Cartwright, K. V., Russell, P., & Kaminsky, E. (2012). Finding the maximum magnitude response (gain) of second-order filters without calculus. Latin-American Journal of Physics Education, 6(4), 559-565. http://www.lajpe.org/dec2012/8_LAJPE_703_Kenneth_Cartwright_preprint_corr_f.pdf
3. Erickson, R. W., & Maksimović, D. (2001). Fundamentals of power electronics (2nd ed.). Springer.
4. Sandler, Steven M (2016). Killing the Bode Plot, DesignCon 2016, Santa Clara CA. https://www.picotest.com/wp-content/uploads/2016/01/Killing-the-Bode-Plot-Final.pdf
5. Sandler, Steven M (2017). Bode Plots are Overrated Signal Integrity Journal https://www.signalintegrityjournal.com/articles/585-bode-plots-are-overrated
6. Nogawa, Masashi 2024. ZOUT to NISM: Output Impedance to Non-Invasive Stability Measurement Microwave Journal, https://www.microwavejournal.com/blogs/32-rf-signal-integrity-to-power-integrity/post/41276-zout-to-nism-output-impedance-to-non-invasive-stability-measurement
7. Sandler, S. M., Janabi, A., Shillaber, L., Zhang, J., Hymowitz, C., & Long, T. (1/19/2025). Non-Invasive Stability Measurement (NISM) by using Non-Invasive Impedance Frequency Analysis. TechRxiv. https://doi.org/10.36227/techrxiv.XXXX (doi.org in Bing)
8. Sandler, S. M. (2023). Quantifying PDN impedance flatness from Sandler NISM. In Proceedings of DesignCon 2023. Santa Clara, CA.
9. Sandler, S. M. (2024). NISM integration in Cadence PSpice: Non-invasive stability measurement in simulation. Picotest Technical Note.
10. Sun, J. (2001). Impedance-based stability criterion for grid-connected inverters. IEEE Transactions on Power Electronics, 16(5), 508–513